Undecidable — <u>NO</u> algorithm

Problems <u>about</u> the behavior of machines/algos.

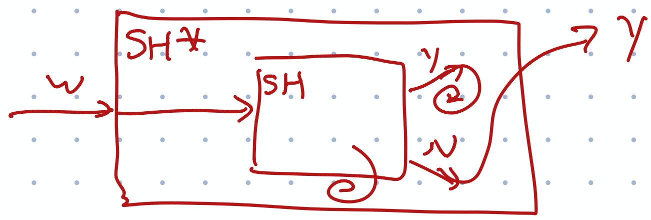
<u>Halting problem:</u> Given code $\langle M \rangle$
and a string w
Does M halt given input w?

<u>SELFHALT:</u> Given $\langle M \rangle$ does M halt on $\langle M \rangle$?

Suppose SH decides SELFHALT   Impossible

ACCEPT(SH) = SELFHALT
REJECT(SH) = $\Sigma^* \setminus$ SELFHALT

<u>SH*(w):</u>
if SH(w) accepts
  hang
else
  accept



ACCEPT(SH*) = REJECT(SH)

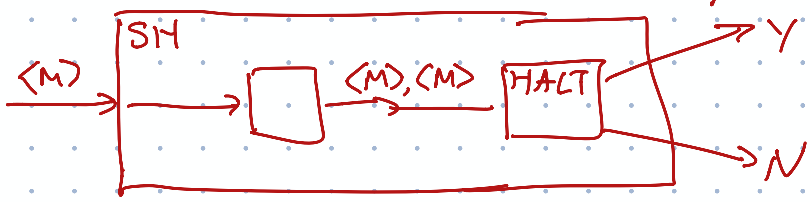
IF SH* accepts $\langle$SH*$\rangle$ $\Rightarrow$ SH accepts $\langle$SH*$\rangle$
$\Rightarrow$ SH* hangs on $\langle$SH*$\rangle$
$\Rightarrow$ SH rejects $\langle$SH*$\rangle$
$\Rightarrow$ SH* accepts $\langle$SH*$\rangle$

HALT is undecidable.

Spose H decides HALT

Write <u>SH(w):</u>
verify w is encoding of some M
return H(w, w)

SH decides SELFHALT — impossible! $\square$



NEVERHALT: Given ⟨M⟩, does M always halt?

Spose NH decides NEVERHALT.

H(⟨M⟩, w):
    Write the following code:
        $M_w(x)$:
            return M(w)
    return ¬NH(⟨M_w⟩)

NH
H
M
M_w

- Suppose M halts on w:
  Then M_w halts on all inputs.
  So NH rejects ⟨M_w⟩
  So H accepts ⟨M⟩, w
- Suppose M hangs on w:
  So M_x hangs on all inputs
  So NH accept ⟨M_w⟩
  So H rejects ⟨M⟩, w

Rice's Theorem

Given ⟨M⟩, does M accept ______?

$$\text{ACCEPT}(M) = \{w \mid M \text{ accepts } w\}$$

Let $\mathcal{L}$ be any set of languages such that
- There is a program Y s.t. $\text{ACCEPT}(Y) \in \mathcal{L}$
- There is a program N s.t. $\text{ACCEPT}(N) \notin \mathcal{L}$

Then deciding if $\text{ACCEPT}(M) \in \mathcal{L}$ for all M is impossible

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Proof (sketch): Assume $\emptyset \notin \mathcal{L}$ $N = M_{reject}$

Suppose Y accepts language in $\mathcal{L}$.

Spose MAGIC decides if $\text{ACCEPT}(M) \in \mathcal{L}$

Build

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H(<M>, w)
    write this code:
        WTF(x):
            call M(w)
            return Y(x)
    return MAGIC(<WTF>)
```

- Does M accept ε?

  $\mathcal{L}$ = languages that contain ε

  Y = accept everything

  N = reject everything

- Does M accept ILLUMINATI ?

- Does M accept <u>only</u> ILLUMINATI?

- Does M accept <u>all</u> palindromes whose length is 2^{prime}?

- Does M accept either $\emptyset$ or Σ^*?

- Does M accept a non-regular language?

  Y = accept all palindromes, nothing else

  N = accept everything