☆ induction on strings

languages → 1 set of T/F

4 open-ended (w/parts)

☆ regular langs/expr

☆ DFAs

product const

NFAs (ε-transitions)

[Subset const
Thompson's]

✓☆ Fooling sets

✓☆ Langunange tranformation
closure properties

✓☆ CFG/CFL

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① $\text{cycle}(L) = \{yx \mid xy \in L\}$

$L = \{\text{STRING}\}$

$\text{cycle}(L) = \{\text{STRING}, \text{TRINGS}, \text{RINGST}, \ldots, \text{GSTRIN}\}$

Prove L is reg $\Rightarrow$ $\text{cycle}(L)$ is reg.

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② $\text{odd}(w)$ = every odd-indexed char

$\text{odd}(\text{STRING}) = \text{SRN}$

Prove L is reg $\Rightarrow$ $\text{odd}(L) = \{\text{odd}(w) \mid w \in L\}$ is reg

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③ $L = \{0^{F_n} \mid n \geq 0\}$ F_n = Fibo #

$F_0 = 0$
$F_1 = 1$
$F_n = F_{n-1} + F_{n-2}$

Prove L is not regular

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④ L = all non palindromes

ⓐ Prove L not regular.

ⓑ Find CFG for L.

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⑤ Transition notation prod/sub

$\{w \mid \text{binary}(w^R) \text{ is div by } 7\}$

DFA for this lang!

① cycle(L) = {yx | xy ∈ L}

L = {STRING}
cycle(L) = {STRING, TRINGS, RINGST, ..., GSTRIN}

Prove L is reg ⇒ cycle(L) is reg.

Let $M = (Q, s, A, \delta)$ be a DFA for L

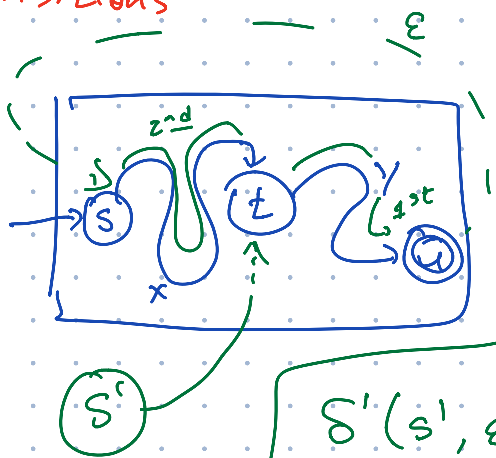
We build an NFA $M' = (Q', s', A', \delta')$ for cycles(L)
(ε-trans)

part of state

~~"Guess x"~~ Guess $\delta^*(s, x) = t$
Read symbols pass them to M starting here

? Guess we're done with y, start reading x.
Read symbols pass them to M starting at s.

ε-transitions



$Q' = Q \times Q \times \{\text{before}, \text{after}\} \cup \{s'\}$

s' is right here

$A' = \{(t, t, \text{after}) \mid t \in Q\}$

$\delta'(s', \varepsilon) = \{(t, t, \text{before}) \mid t \in Q\}$

$\delta'((t, q, \text{before}), \varepsilon) = \begin{cases} \{(t, s, \text{after})\} & \text{if } q \in A \\ \emptyset & \text{otherwise} \end{cases}$

$\delta'((t, q, \text{whatever}), a) = \{(t, \delta(q, a), \text{whatever})\}$

Anything else $\longrightarrow \emptyset$

(3) $L = \{0^{F_n} \mid n \geq 0\}$ $F_n = $ Fibo #

$F_0 = 0$
$F_1 = 1$
$F_n = F_{n-1} + F_{n-2}$

$L = \{\varepsilon, 0, 00, 000, 00000, 00000000, \ldots\}$

Prove L is not regular

Let $F = 0^*$

Let x and y be arb. string in F

then $x = 0^i$ and $y = 0^j$ for some $i \neq j$ wlog $i < j$

Let $z = 0^k$ where $k = F_n - i$ and $n = i + j + 100001$

Then $xz = 0^i 0^k = 0^{i+k} = 0^{F_n}$

$yz = 0^j 0^k = 0^{j+k} = 0^{F_n + j - i}$

because $F_n + j - i < F_{n+1}$

$j - i < F_{n-1}$

So F is a fooling set. F is infinite, so L can't be reg $\square$

$j - i < 100000 < F_{100000} < F_{j-i+100000}$

or

$j - i < F_{j-i} < F_{j-i+100000}$

(3) $L = \{0^{F_n} \mid n \geq 0\}$ $\quad F_n = \text{Fibo \#}$

$F_0 = 0$
$F_1 = 1$
$F_n = F_{n-1} + F_{n-2}$

Prove L is not regular

Let $F = L \setminus \{\varepsilon, 0, 00, 000, 00000, 00000000\}$

Choose $x \neq y$ from F

then $x = 0^{F_n}$ $\quad$ wlog $m < n$

$y = 0^{F_m}$ $\quad \boxed{m > 6,\ n > 6}$

Let $z = 0^{F_{n+1}}$

Then $xz = 0^{F_n} 0^{F_{n+1}} = 0^{F_{n+2}} \in L$

$yz = 0^{F_m} 0^{F_{n+1}} = 0^{F_{n+1} + F_m} \notin L$

$> F_{n+1}$ $\quad < F_{n+2}$

(4) L = all <u>non</u> palindromes

(a) Prove L not regular.

(b) Find CFG for L.

(a) Let $F = \{0^n 1^n \mid n \cancel{\geq 0}\ {>}374\}$

Let x and y be any strings in F

Then $x = 0^i 1^i$ for some $i<j$ (wlog)

$y = 0^j 1^j$

Let $z = 0^i$

Then $xz = 0^i 1^i 0^i \notin L$ palindrome

$yz = 0^j 1^i 0^i \in L$ not a pal. because $i \neq j$

So F is an infinite fooling set.

(a) Let $F = 0^*$

Let x and y be any strings in F

Then $x = 0^i$

$y = 0^j$

Let $z = 10^i$

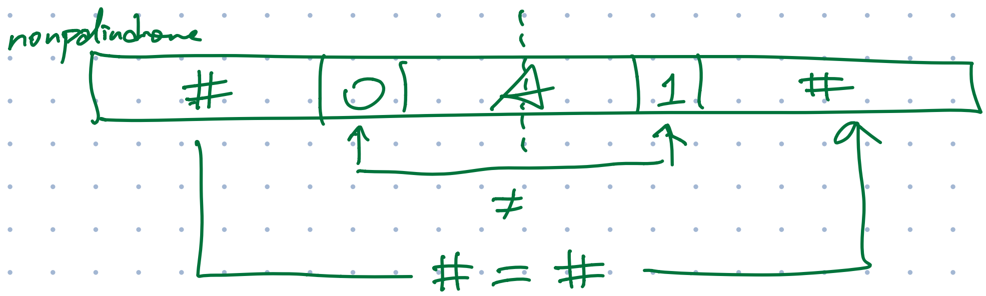
Then $xz = 0^i 1 0^i \in L$

$yz = 0^j 1 0^i \notin L$

So F is an infinite fooling set.

(4) L = all non palindromes
(a) Prove L not regular.
(b) Find CFG for L.

Palindromes:
$S \to \varepsilon \mid 0 \mid 1 \mid 0S0 \mid 1S1$



$S \to M \mid \cancel{0S1} \mid 0S0 \mid \cancel{1S0} \mid 1S1$ non-palindromes
$M \to 0A1 \mid 1A0$ middle - first ≠ last
$A \to 0A \mid 1A \mid \varepsilon$ any string

A, B are CFL

$A \cup B$ is CF $S \to A \mid B$

$A \circ B$ is CF $S \to AB$

A^* is CF $S \to \varepsilon \mid AS$

$A \cap R$ is CF
↑ regular

$\{0^n 1^n 0^n \mid n \geq 0\}$ is not CF

$S \to AB$
$A \to 0A \mid \varepsilon$
$B \to 1B0 \mid \varepsilon$

$S \to 010S \mid \varepsilon$

⑤ DFA for this lang.

$\{w \mid binary(w^R) \text{ is div by } 7\}$

1	1
10	2
100	4
1000	1
10000	2
100000	4
1000000	1

binary(w) is div by 7:

$binary(\varepsilon) = 0$

$\boxed{binary(x \cdot a) = 2 \cdot binary(x) + a}$

$\delta(q, a) = 2q + a \mod 7$

$Q = \{0, 1, 2, \dots, 6\} \times \{0, 1, 2, 3, \dots 6\}$

$s = (0, 1)$

$A = \cancel{\{(0,1), (0,2), (0,4)\}}\ A \times \{0 \dots 6\}$

$\delta((r, p), a) = ((r + p \cdot a) \mod 7,\ 2p \mod 7)$

r — remainder

p - phase