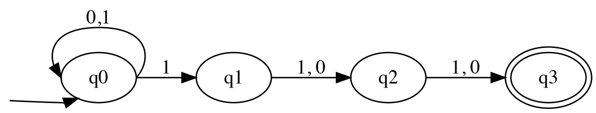
CS/ECE 374 B — Lab 3 — Fall 2019
Solutions

1. Construct an NFA that accepts all binary strings that have a 1 as the third last symbol. I.e., 0000100 and 1111 are in the language, 1010 is not.



Solution:

2. Given an NFA $N = (\Sigma, Q, \delta, s, A)$, construct an NFA N' that accepts all *prefixes* of $L(N)$, i.e., $w \in L(N') \Leftrightarrow wx \in L(N)$ for some $x \in \Sigma^*$.

Solution: Let S be the set of states in Q that are both reachable from s and can reach an accepting state.[1] Make every state in S accepting; i.e., $N' = (\Sigma, Q, \delta, s, S)$.

3. Given an NFA $N = (\Sigma, Q, \delta, s, A)$, construct an NFA N' that accepts all *suffixes* of $L(N)$, i.e., $w \in L(N') \Leftrightarrow xw \in L(N)$ for some $x \in \Sigma^*$.

Solution: Let S be the set of states in Q that are both reachable from s and can reach an accepting state. Create a new start state s_0 and create an ϵ-transition from s_0 to every state in S.

I.e., $N' = (\Sigma, Q \cup \{s_0\}, \delta', s_0, A)$ where:

$$\begin{aligned} \delta'(s_0, \epsilon) &= S \\ \delta'(s_0, c) &= \emptyset && \text{for any } c \in \Sigma \\ \delta'(s, c) &= \delta(s, c) && \text{for any } c \in \Sigma \cup \{\epsilon\}, s \in Q \end{aligned}$$

Note that the addition of the extra state s_0 is necessary to avoid the ϵ transition being taken after the NFA takes a series of steps and returns to s.

4. Given an NFA $N = (\Sigma, Q, \delta, s, A)$, construct an NFA N' that accepts $\mathsf{insert1}(L(N))$, i.e., strings from $L(N)$ with a $\mathtt{1}$ inserted somewhere. In other words, $x \in L(N')$ if $x = y\mathtt{1}z$ and some $y, z \in \Sigma^*$ and $yz \in L(N)$.

Solution: We can essentially have two copies of the state Q, with a transition between them when we see a 1. Formally:

$$\begin{aligned} N' &= (\Sigma, Q \times \{0,1\}, \delta', (s,0), A') && \text{where} \\ A' &= \{(q,1) | q \in A\} \\ \delta'((q,0), \mathtt{1}) &= \{(r,0) | r \in \delta(q, \mathtt{1})\} \cup \{(q,1)\} && \text{for } q \in Q \\ \delta'((q,0), c) &= \{(r,0) | r \in \delta(q,c)\} && \text{for } q \in Q, c \in \Sigma \cup \{\epsilon\} - \{\mathtt{1}\} \\ \delta'((q,1), c) &= \{(r,1) | r \in \delta(q,c)\} && \text{for } q \in Q, c \in \Sigma \cup \{\epsilon\} \end{aligned}$$

5. Given an NFA $N = (\Sigma, Q, \delta, s, A)$, construct an NFA N' that accepts the reverse of $L(N)$, i.e., $w \in L(N') \Leftrightarrow w^R \in L(N)$.

Solution: This can be done by reversing every transition, and adding an extra starting state s_0 with an ϵ-transition to every accepting state. The original start state s becomes the sole accepting state.

$N' = (\Sigma, Q \cup s_0, \delta', s_0, \{s\})$ where:

$$\begin{aligned} \delta'(s_0, \epsilon) &= A \\ \delta'(s_0, c) &= \emptyset && \text{for any } c \in \Sigma \\ \delta'(s, c) &= \{t \in Q | s \in \delta(t, c)\} && \text{for any } c \in \Sigma \cup \{\epsilon\}, s \in Q \end{aligned}$$