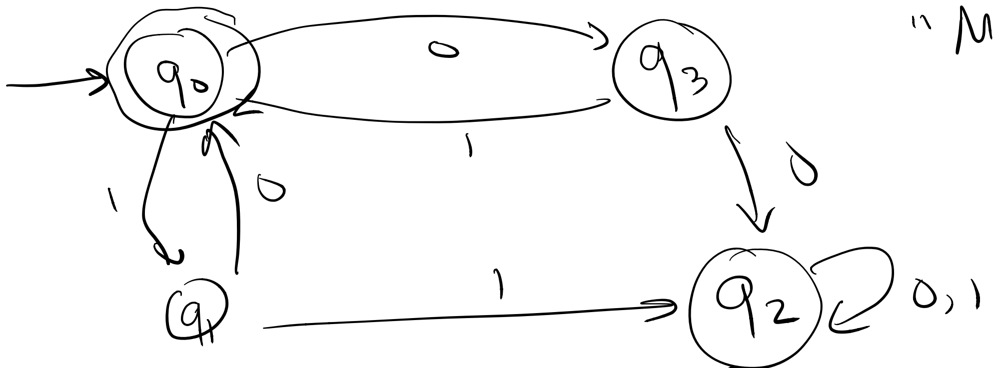
Lecture 4: DFAs

Formal def'n Product construction

Midterm 1: Sep 30, 7-9 pm
2: TBA
Final (tentative): , Dec 17 (Tue) 1:30-4:30

$\delta(q_0, 1) = q_1$

"M"



M accepts a string w if (unique) walk starting at start state and following transitions based on symbols of w ends in an accepting state

1 0 0 1

$q_0\ q_1\ q_0\ q_3\ q_0$

$(01 | 10)^*$

Σ - alphabet $\qquad \{0, 1\}$

Q - states $\qquad \{q_0, q_1, q_2, q_3\}$

$s \in Q$ - start $\qquad q_0$

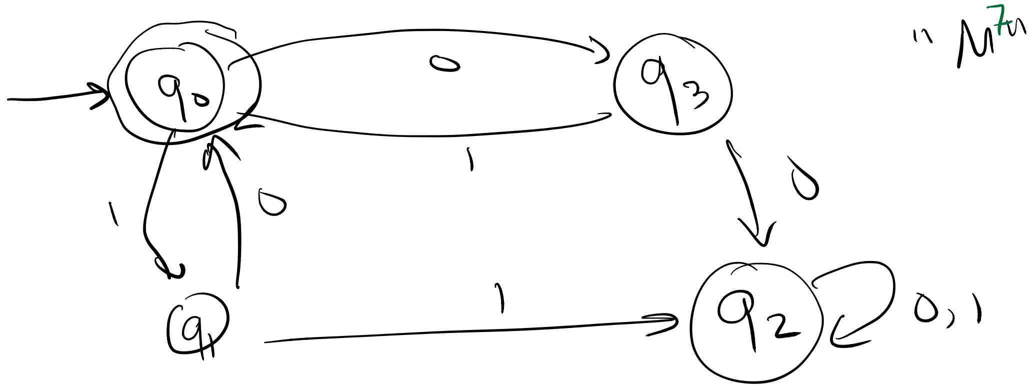
$A \subset Q$ - accepting states $\qquad \{q_0\}$

$\delta : Q \times \Sigma \to Q$

transition function

in state δ	in char	new state
q_0	1	q_1
q_0	0	q_2
q_1	1	q_3
q_1	0	q_0

DFA $M = (\Sigma, Q, \underset{q_0}{s}, \underset{F}{A}, \delta)$



1 0 0 1

- $s = q_0$
- $\delta(q_0, 1) = q_1$
- $\delta(q_1, 0) = q_0$
- $\delta(q_0, 0) = q_3$
- $\delta(q_3, 1) = q_0$

1001 is accepted by $M = (\Sigma, Q, s, A, \delta)$

$\equiv \underline{\delta(\delta(\delta(\delta(s, 1), 0), 0), 1) \in A}$

Given δ define $\delta^*: Q \times \Sigma^* \to Q$

$$\delta^*(q, \epsilon) = q$$
$$\delta^*(q, ax) = \delta^*(\delta(q,a), x)$$

$$\delta^*(s, 1001) = \delta(\delta(\delta(\delta(s,1),0),0),1)$$

M accepts $w \equiv \delta^*(s, w) \in A$

L(M) is language accepted by DFA $M = (\Sigma, Q, s, A, \delta)$

$$L(M) = \{ w \in \Sigma^* \mid \delta^*(s, w) \in A \}$$

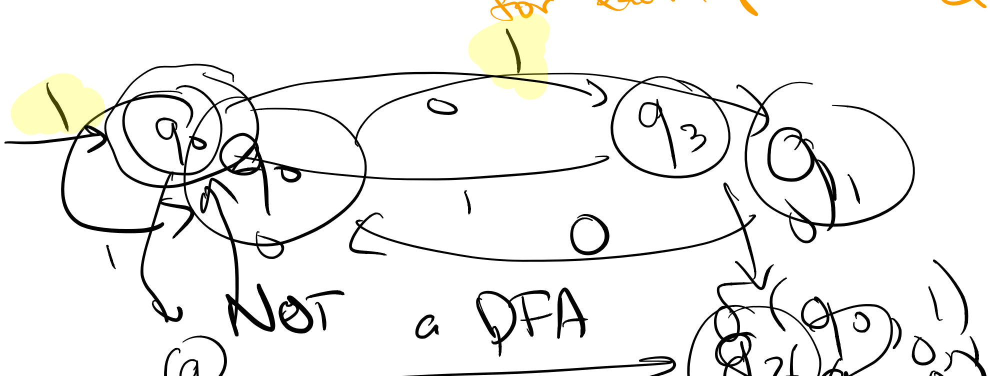
$$L(M) = (01+10)^*$$

our running example DFA

"D" in DFA $\rightarrow$ δ is a total function

defined for all pairs in $Q \times \Sigma$

single value for each input



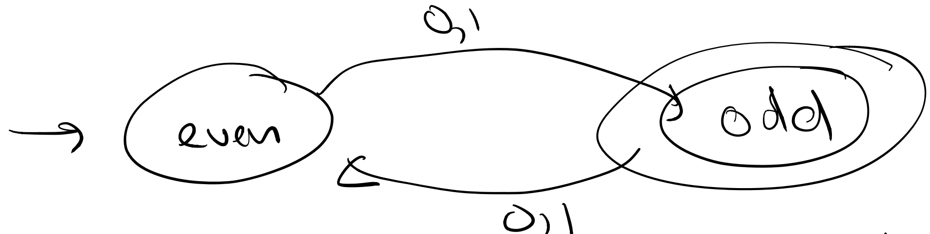
NOT a DFA $\delta(q_0, 1) = ??$

reject state

implicit reject state "r"

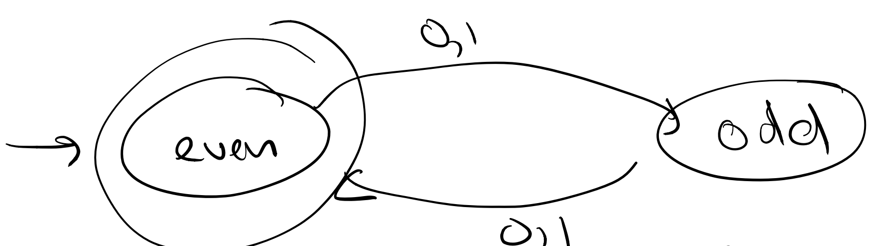
$\delta(q, c) = r$

for any non-labeled trans.



$L_{odd} =$ odd length strings

$L_{even} = \Sigma^* - L_{odd} = \overline{L_{odd}}$



$M = (\Sigma, Q, s, A, \delta)$

$M' = (\Sigma, Q, s, Q-A, \delta)$

→ Lang's accepted by DFAs closed under complement

(if a DFA accepts L then <u>some</u> other DFA accepts $\Sigma^* - L = \overline{L} = L^c$)

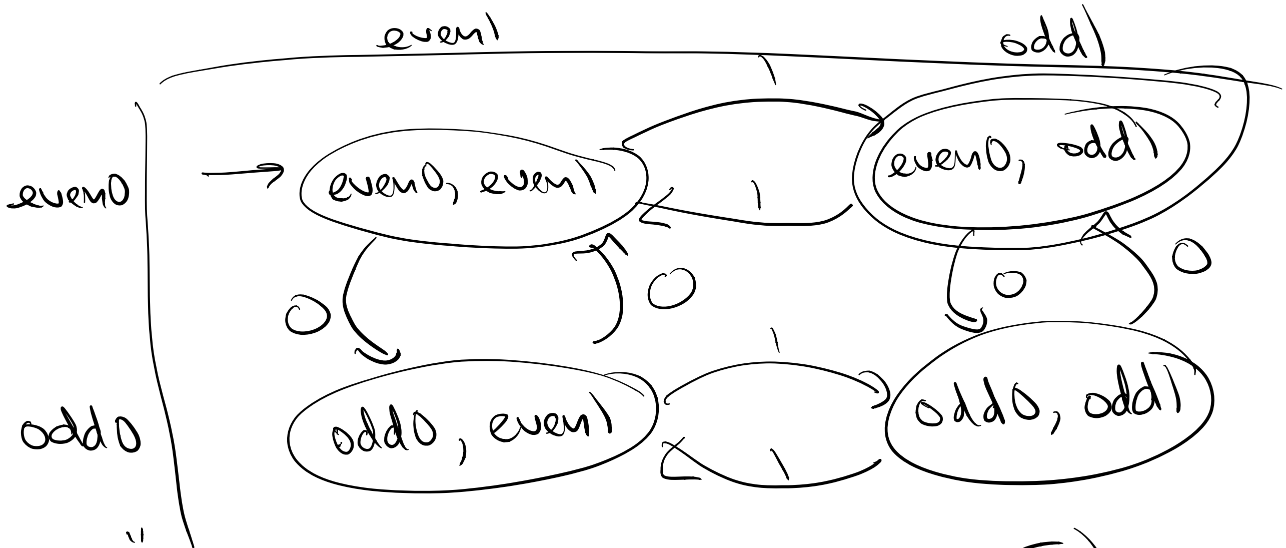
$L_1 = L(M_{odd1})$

$L_2 = L(M_{even0})$

$L_3 = L_1 \cap L_2$

accept (dfa, string) : true/false

accept_L3 (string) :
return accept(L1, string) <u>and</u> accept(L2, string



$$M_1 = (\Sigma, Q_1, s_1, A_1, \delta_1)$$
$$M_2 = (\Sigma, Q_2, s_2, A_2, \delta_2)$$

$\cancel{A}$ $Q = Q_1 \times Q_2$ $\quad Q_1 \not\subset Q$

$s = (s_1, s_2)$

$A = \{ (q_1, q_2) \in Q_1 \times Q_2 \mid q_1 \in A_1 \text{ and } q_2 \in A_2 \}$

$A_1 \times A_2$

$$\delta((q_1, q_2) \in Q_1 \times Q_2, a \in \Sigma) = (\delta_1(q_1, a), \delta_2(q_2, a))$$

<u>Theorem</u>: $M = (Q, \Sigma, s, A, \delta)$ as above

$$L(M) = L(M_1) \cap L(M_2)$$

<u>Proof</u>

<u>Lemma 1</u> $\delta^*((q_1, q_2), w) = (\delta_1^*(q_1, w), \delta_2^*(q_2, w))$

<u>Proof</u> by induction

$$w \in L(M) \iff \delta^*((s_1, s_2), w) \in A$$

$$\iff (\delta_1^*(s_1, w), \delta_2^*(s_2, w)) \in A \quad \text{(by Lemma)}$$

$$\iff \delta_1^*(s_1, w) \in A_1 \text{ and } \delta_2^*(s_2, w) \in A_2$$

$$\iff w \in L_1 \text{ and } w \in L_2$$

$$Q_1 = \{q_0, \underline{q_1}\}$$

$$Q_2 = \{\underline{q_0}, q_1\}$$

$$Q = Q_1 \times Q_2$$

$$= \{(q_0, q_0), (q_0, q_1), \boxed{(q_1, q_0)}, (q_1, q_1)\}$$