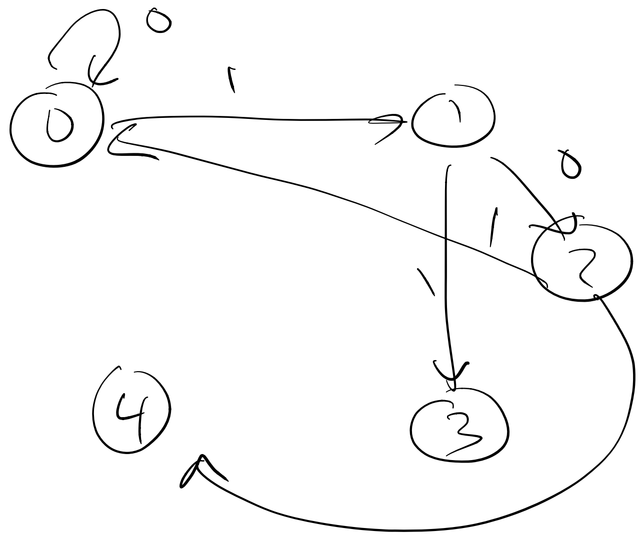


$$\delta(i, '0') = i \cdot 2 \mod 5$$
$$\delta(i, '1') = i \cdot 2 + 1 \mod 5$$

Non-deterministic finite automata

"D" in DFA $\iff$ δ is a total function

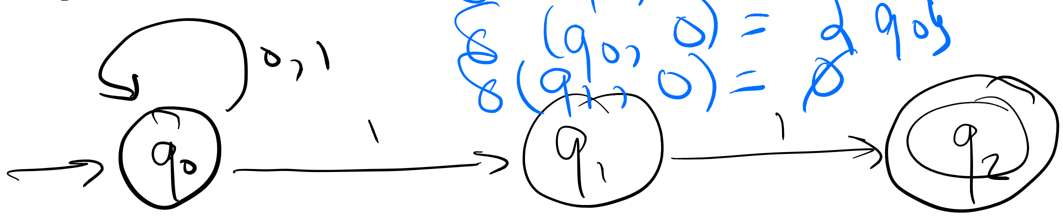
δ is uniquely defined for each pair in $Q \times \Sigma$

non-DFA

$\delta(q_0, 1) = \{q_0, q_1\}$

$\delta(q_0, 0) = \{q_0\}$

$\delta(q_1, 0) = \emptyset$



any missing transition → reject

"when you come to a fork in the road, take it"

0 1 0

q_0 q_0 q_0 q_0

q_0 q_0 q_1 X

0 1 1

q_0 q_0 q_1 q_2

q_0 q_0 q_0 q_1

In DFA there's a unique walk for any input string

In an NFA there are 0 or more walks for each string

An NFA accepts string w if **some** walk starting at s ends in accepting state.

$$DFA = (\Sigma, Q, \delta, s, A)$$

$$NFA = (\Sigma, Q, \delta, s, A)$$

$$\delta : Q \times \Sigma \to \underline{2^Q} \quad P(Q)$$

$$\delta^* : Q \times \Sigma^* \to 2^Q$$

$$\delta^*(q, \epsilon) = \{q\}$$

$$\delta^*(q, ax) = \bigcup_{p \in \delta(q,a)} \delta^*(p, x)$$

$$\delta(q_0, 1) = \{q_0, q_1\}$$

$$\delta(q_0, 0) = \{q_0\}$$

$$\delta(q_1, 0) = \emptyset$$

$$\delta^*(q_0, 01) = \bigcup_{p \in \delta(q_0, 0)} \delta^*(p, 1)$$

$\hat{\delta}(q_0, 011) = \{q_0, q_1, q_2\}$

$\hat{\delta}(q_0, 010) = \{q_0\}$

$= \hat{\delta}(q_0, 1)$

$= \bigcup_{p \in \delta(q_0,1)} \hat{\delta}(p, \epsilon)$

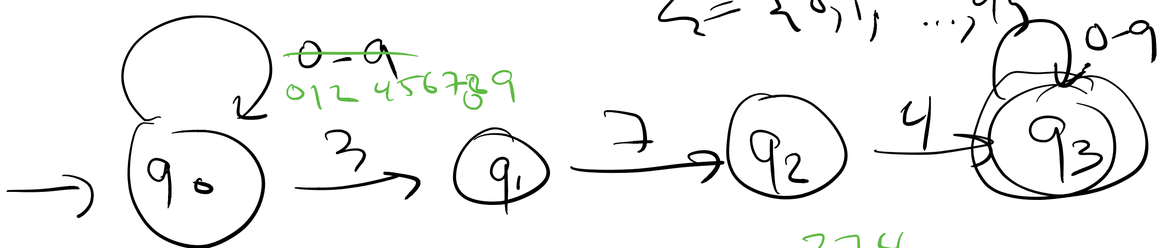
$= \bigcup_{p \in \{q_0, q_1\}} \hat{\delta}(p, \epsilon)$

$= \{q_0, q_1\}$

NFA N accepts w if

$\hat{\delta}(s, w) \cap A \neq \emptyset$

$\Sigma = \{0, 1, \ldots, 9\}$



374
13749

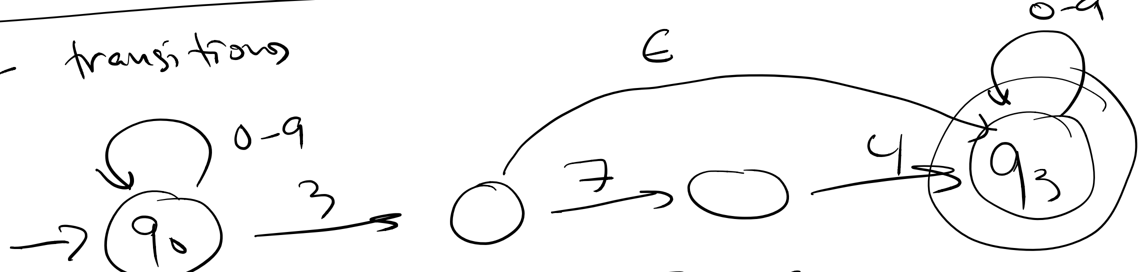
137 259 3741

green M $q_0 q_0 q_1 q_2$ X

original N $q_0 q_0 q_1 q_2$ X

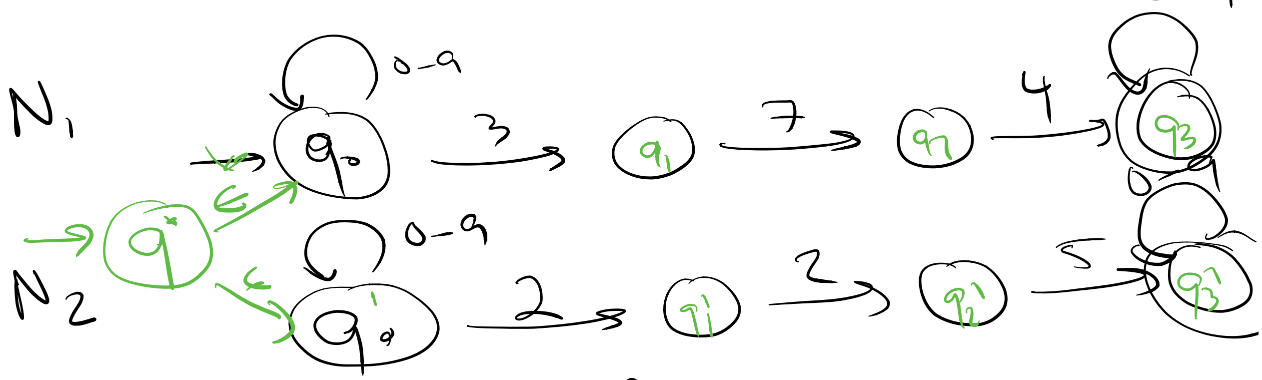
$q_0 q_0 q_0 q_0 q_0 \ldots q_1 q_2 q_3$

ϵ - transitions



$q_0 \overset{1}{} q_0 \overset{2}{} q_0 \overset{3}{} q_1 \overset{\epsilon}{} q_3$

$$\delta : Q \times (\Sigma + \{\epsilon\}) \to 2^Q$$



ϵ-reach : $Q \to 2^Q$

the set of all states reachable using <u>only</u> ϵ transitions

ϵ-reach q^* : $\{q^*, q_0, q_0'\}$

$$\delta^*(q, \epsilon) = \epsilon\text{-reach}(q)$$

$$\delta^*(q, ax) = \bigcup_{p \in \epsilon\text{reach}(q)} \left(\bigcup_{r \in \delta(p,a)} \delta^*(r,x) \right)$$

$$\delta^*(q^*, 3x) = \bigcup_{p \in \{q^*, q_0, q_0'\}} \left(\bigcup_{r \in \delta(p,3)} \delta^*(r,x) \right)$$

$$= \bigcup_{r \in \delta(q^*,3) \cup \delta(q_0,3) \cup \delta(q_0',3)} \delta(r,x)$$

q_0, q_0', q_1