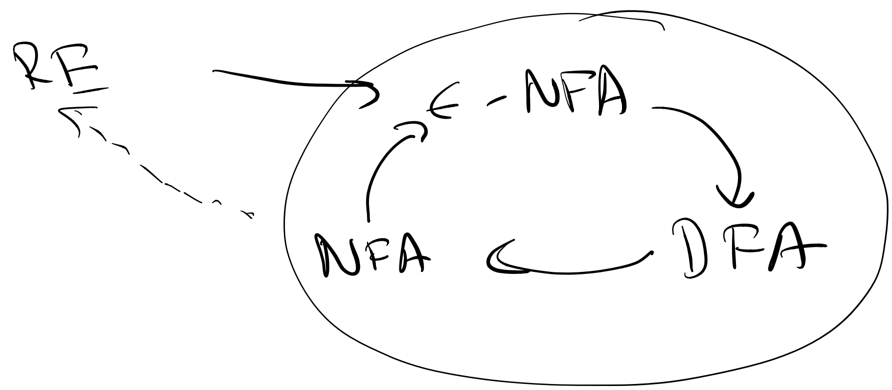
Today

1. DFA → RE
2. Proving non-Regularity
   - fooling sets
   - closure properties

RE → ϵ-NFA

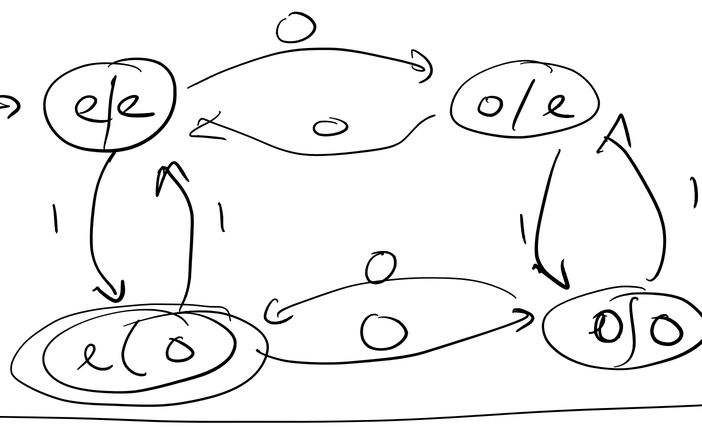
ϵ-NFA → DFA

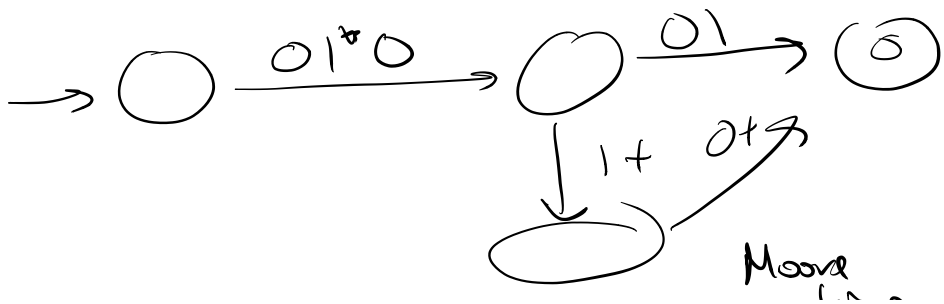
(ϵ-NFA → NFA)



M_{eo}

$L_{eo} = L(M_{eo})$



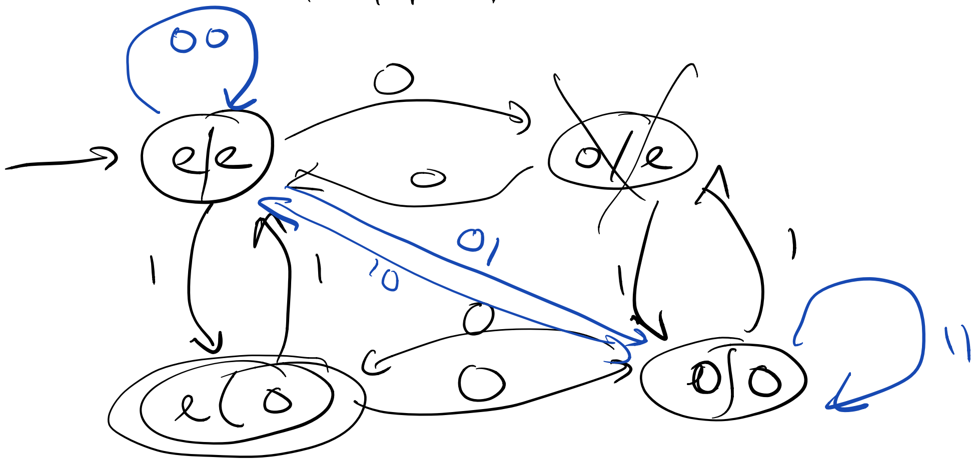


Moore

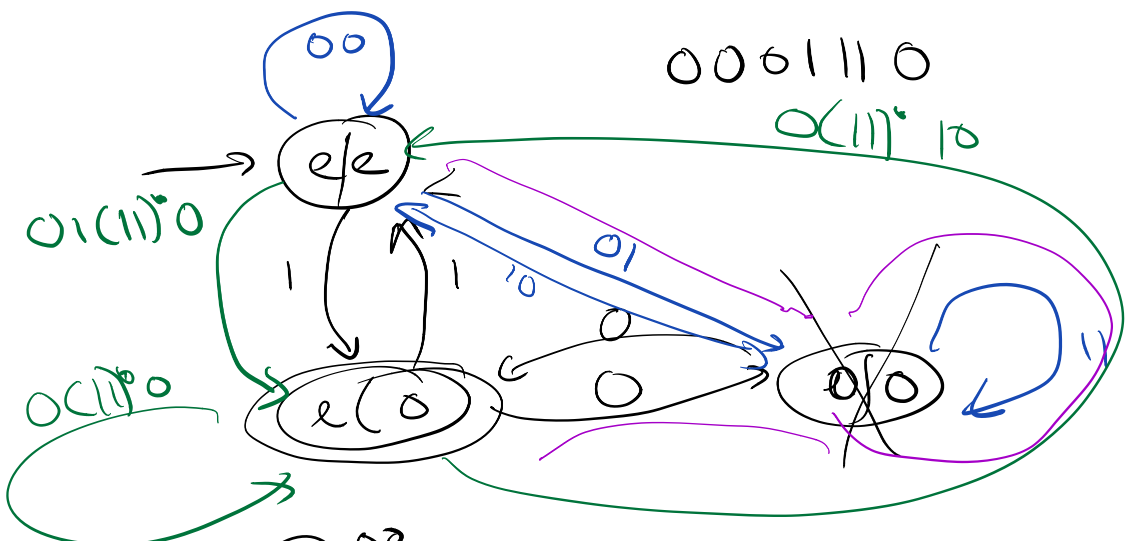
machine

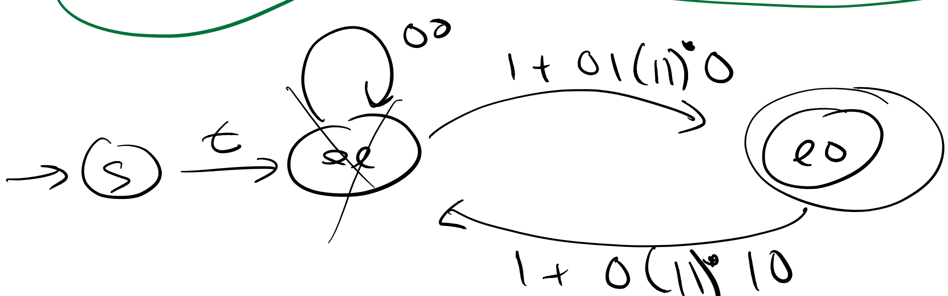
$0\,1^*\,0\,0\,1$

$0\,1^*\,1+0+$



$000111\,0$





$$\rightarrow (S)\quad (00)^*(1+01(11)^*0)\big((1+0(11)^*10)(00)^*(1+01(11)^*0) + (0(11)^*0)\big)^*$$

Closure

RLs are closed under

- → concatenation
- → union
- → Kleene *
- → complement
- → intersection

→ set difference

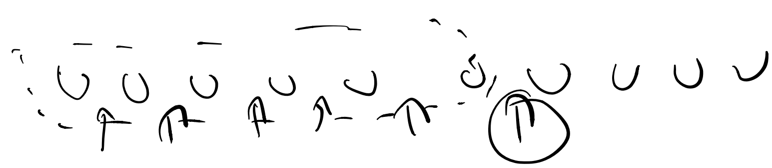
. . .

| languages | uncountably infinite

re $\in \left(\Sigma + \{ \emptyset, \epsilon, ^*, +, (,) \} \right)^*$

q1#q2#q3##q3##q1#a#q2#...

DFA({ "even", "1", "odd", ... })



Q

$\delta^*(s_0, x)$ y

$\mapsto \delta^*(s_0, xy)$

$L_P = \{ w \mid w = w^R \}$

$w = xy \qquad |x| = |y|$

$\underbrace{\quad x \quad}_{\delta^*(s_0, x)} \; \underbrace{\quad y \quad}_{y = x^R}$

$\underbrace{ab\,c\,def}_{?}\ \underbrace{fed\,cba}$

$L = \{0^n 1^n \mid n \geq 0\}$

$00000000\,0\,0\ 11\,11\,1111$

8 9 ?

L $\quad x \equiv_L y$ if for all w

$xw \in L \iff yw \in L$

L = even length strings

$00 \equiv_L 0000$

$00\,0 \notin L \quad 0000\,0 \notin L$

$00\,00 \in L \quad 0000\,00 \in L$

Lemma 1: if $x \not\equiv_L y$ then DFA M with $L = L(M)$ $\delta^*(s,x) \neq \delta^*(s,y)$

Proof $\exists w$ st $xw \in L$ and $yw \notin L$ (or vice versa)

$\delta^*(s, xw) = \delta^*(\delta^*(s,x), w)$

if $\delta^*(s,x) = \delta^*(s,y)$ then

$A \ni \delta^*(s, xw) = \delta^*(\delta^*(s,x), w) =$

$\delta^*(\delta^*(s,y),w) = \delta^*(s,yw) \notin A$

$\#$

Def For L, a fooling set F has as property $x, y \in F$ $x \neq y$

$$x \not\equiv_L y$$

"set of pairwise distinguishable strings"

Cor L, F a fooling set for L, DFA $M = (\Sigma, Q, \delta, s, A)$ accepts L

then $|Q| \geq |F|$

L_{oe} odd 0's, even 1's

$$\{1, 0, 10, \epsilon\}$$

$w=1$ $\quad$ $w=1$

$01 \notin L$ $\quad$ $101 \in L$ $\quad$ $101 \in L$ $\quad$ $1 \notin L$

Cor If L has an infinite fooling set, L is not regular.

$$L_1 = \{0^n 1^n \mid n \geq 0\}$$

$$F_1 = \{0^n \mid n \geq 0\}$$

$x, y \in F_1$ $\quad$ $x \not\equiv_{L_1} y$

$x \neq y$.

$x = 0^i \quad y = 0^j \quad i \neq j$

$x 1^i \in L \quad y 1^i = 0^j 1^i \notin L$

$L_2 = \{ w \mid w = w^R \}$

$F_2 = \{ 0^n 1 \mid n \geq 0 \}$

$x, y \in F_2 \quad x = 0^i 1 \quad i \neq j$
$y = 0^j 1$

$0^i 1 w \in L \quad w = 0^i$
$0^j 1 w \notin L \quad w = 1 0^i$

$F_2' = \Sigma^*$

$x, y \in \Sigma^* \quad w = x^R$

$xw \in L \quad yw \notin L \quad y \neq x$
not quite true

$x = 01$
$y = 010$

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1. Myhill-Nerode
DFA w/ min states for L
has $|Q| =$ largest fooling set.

Loe max $|F| = 4$

2. $L_3 = \{w \mid w$ has same # of 1's and 0's $\}$

$L_4 = L_3 \cap 0^* 1^* = L_1 = \{0^n 1^n\}$

if L_3 regular $\Rightarrow L_1$ is regular

L_1 not regular $\Rightarrow L_3$ is not regular

$L_1 = L_3 \cap \underline{0^* 1^*}$

regular $\rightarrow$

$\exists M_0$ accepts $0^* 1^*$

suppose L_3 is regular

$\exists M_3$ accepts L_3

$\Rightarrow \exists M'$ accepts $L_3 \cap 0^* 1^*$

by the produ-

$\Rightarrow L_1$ is regular #