Lecture 8:
Context-Free Languages (CFLs)

Exam 1: Sep 30 7pm - 9pm
covers up to and <u>including</u> today

$S \to$ SubVerbObj

Sub $\to$ NounPhrase

NounPhrase $\to$ Adj NounPhrase | Noun

Noun $\to$ student | 374 | class | computer

Verb $\to$ likes | swims | shoots

Obj $\to$ NounPhrase

Adj $\to$ blue | excited | new

student likes 374

blue computer swims class

$$\boxed{S \to \epsilon \mid 0 S 1}$$

non-terminals ϵ 0 1

$V = \{S\}$

$T = \{0, 1\}$

$P = \{S \to \epsilon, S \to 0S1\}$

$s = S$

$S \to 0S1 \to 0\epsilon 1 = 01$

$S \to 0\underline{S}1 \to 00\underline{S}11 \to 00\epsilon 11 \to 0011$

Context-Free Grammar

1. Terminal symbols (Σ) = alphabet (T) = constants
2. Non-terminal symbols (V)

= variables $\sim$ states

3. Productions (P)

Rules $V \to TVT \ldots$

$A \in V \quad \alpha \in (V \cup T)^*$

$A \to \alpha$

$P \subset V \times (V \cup T)^*$

4. $S \in V$ starting non-terminal

<u>Define</u> $\alpha \in (V \cup T)^*$ <u>derives</u> $\beta \in (V \cup T)^*$

$$\alpha \rightsquigarrow \beta$$

if

$$\alpha = \gamma N \delta$$

$$\beta = \gamma \eta \delta$$

$$(N \to \eta) \in P$$

$$S \to \epsilon \mid 0 S 1$$

$$\underline{S} \rightsquigarrow \underline{\epsilon}$$

$$\underline{S} \rightsquigarrow \underline{0 S 1}$$

$$00\underline{S}11 \rightsquigarrow 00\underline{\epsilon}11$$

$$\underline{S} \rightsquigarrow 0S1 \rightsquigarrow 00S11 \rightsquigarrow 00\epsilon 11 = \underline{0011}$$

$$S \rightsquigarrow^3 0011$$

$$S \rightsquigarrow^* 0011$$

$$G = (V, T, P, S)$$

$$L(G) = \{ x \in T^* \mid \underbrace{S \rightsquigarrow^* x}\}$$

all x strings over alphabet T ~~that~~ can be derived in some # of steps from S

$$L_3 = \{ x \in \{0,1\}^* \mid \#(0, x) \text{ is odd}\}$$

$$G \equiv \underline{S \to \epsilon \mid 0S1}$$

$$G = (\{0,1\}, \{S\}, \{S \to \epsilon, S \to 0S1\}, S)$$

$$L(G) = \{\epsilon, 01, 0011, 000111, \ldots\}$$

$$\{0^n 1^n \mid n \geq 0\}$$

Prove: (a) $x \in L(G) \to x \in 0^n 1^n$

(b) $x = 0^n 1^n \to x \in L(G)$

$x \in L(G)$ $\quad$ $\boxed{S \rightsquigarrow \alpha_1 \rightsquigarrow \alpha_2 \ldots \alpha_k \rightsquigarrow \boxed{x}}$

~~at least~~ one non-terminal

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for any $k \geq 0$,

$$S \rightsquigarrow^k \alpha \quad \text{means}$$

$$\alpha = 0^i S 1^i$$

or

$$\alpha = 0^i 1^i$$

Assume holds for all smaller k

$$S \rightsquigarrow^k \alpha$$

if $k = 1$ $\quad S \rightsquigarrow \alpha$

$$\Rightarrow \alpha = 0S1 \quad \text{or} \quad \alpha = \epsilon = 0^0 1^0$$

if $k > 1$ $\quad S \rightarrow^{k-1} \alpha' \rightarrow \alpha$

by IH $\quad \alpha' = 0^i S 1^i$

$$\Rightarrow \alpha = 0^{i+1} S 1^{i+1}$$

$$\text{or } \alpha = 0^i \epsilon 1^i$$

$$\alpha' = 0^i 1^i \quad \#$$

no non-terminals

$x = 0^i 1^i$ then $x \in L(G)$

For any $k \geq 0$ $\quad S \rightsquigarrow^* 0^k S 1^k$

$k = 1$ this $S \rightsquigarrow 0S1$

assume true for smaller k

$$S \rightsquigarrow^* 0^{k-1} S 1^{k-1}$$
$$\rightsquigarrow 0^{k} S 1^{k}$$

$$S \to \epsilon = 0^0 1^0$$

$$S \to \epsilon \mid (S) \mid SS$$

$$S \rightsquigarrow \epsilon$$
$$S \rightsquigarrow (S) \rightsquigarrow ()$$
$$S \rightsquigarrow SS \rightsquigarrow (S)S$$
$$\rightsquigarrow ()S \rightsquigarrow ()(S)$$
$$\rightsquigarrow ()()$$

())(

$$S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$$

$$S \rightsquigarrow \epsilon$$
$$S \rightsquigarrow 1$$
$$S \rightsquigarrow 0S0 \rightsquigarrow 01S10$$
$$\rightsquigarrow 01010$$

$$S \to E$$
$$E \to 1E \mid 0O \mid \epsilon$$
$$O \to 1O \mid 0E$$

$S \leadsto E \leadsto 1E \leadsto \underline{1\emptyset 0}$

$\leadsto 1\emptyset 1 0$

$\leadsto 1\emptyset 1\emptyset E$

$\leadsto 1\emptyset 1\emptyset \in L$

$S \leadsto E \leadsto 1E \leadsto 1 \in L$

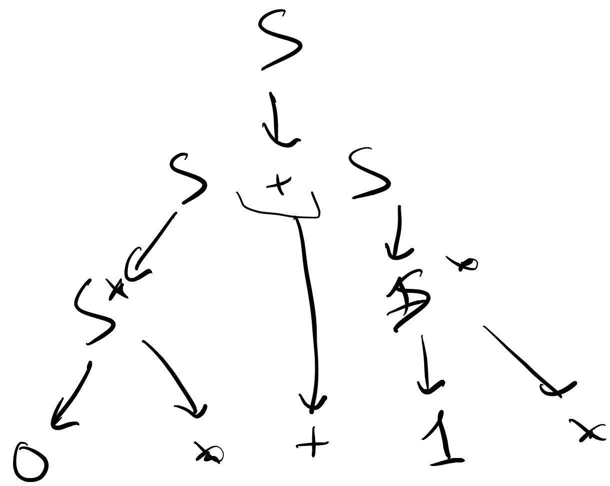
$S \leadsto E \leadsto \emptyset 0 \leadsto \emptyset\emptyset E$

$\leadsto \emptyset\emptyset \in L$

even # 0's

$S \to \emptyset \mid \varepsilon \mid 0 \mid 1 \mid S+S \mid$
$SS \mid S^* \mid (S)$

$T = \{0, 1, \emptyset, \varepsilon, +, *, (,)\}$



parse tree

$S \leadsto 0^* + 1^*$

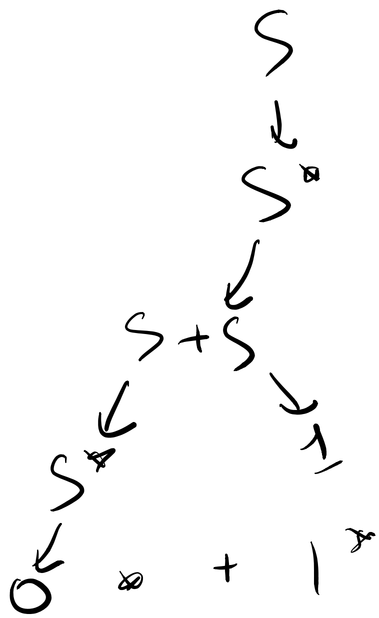
$P \to S \mid S;P$

$S \to V = E \mid$
$F \mid C \mid \ldots$

$V \to \ldots$

$E \to E + E$
$E * E$

$0^* + 1^*$


S
S*
S + S
S*
1
0 * + 1*