Today

- Quick Select
- Binary Search
- Multiplication (Karatsuba)

[Backtracking]

Quick Sort(list)

select pivot

split list into ($<$ pivot) X] $\Theta(n)$

($\geq$ pivot) Y

Quick Sort (X)

Quick Sort (Y)

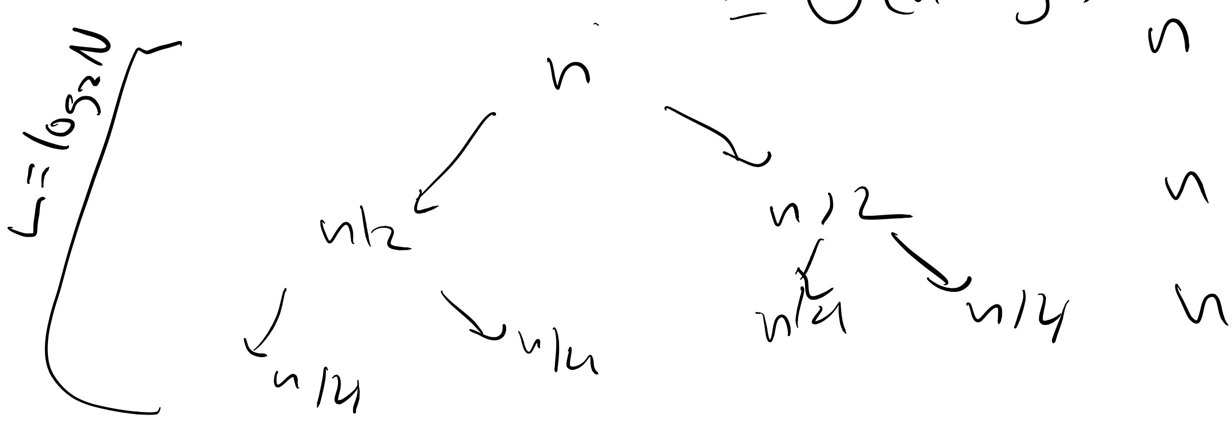
return X + {pivot} + Y

$\Theta(n^2)$ (worst case)

$$T(n) = 2T(n/2) + \Theta(n)$$

best case quick sort

$$= \Theta(n \log n)$$



$$\sum_{i=1}^{L} n = L \cdot n = n \log_2 n$$

$$\text{pivot} = \underset{\text{of } 3}{\text{median}}\left(x[1],\ x[n/2],\ x[n] \right)$$

$$\underset{\text{of } 4}{\text{median}}\left(x[1],\ x[n/3],\ x[2n/3],\ x[n] \right)$$

Quick Sort(list) ← length n

select pivot

split list into ($<$ pivot) X

($\geq$ pivot) Y

m = length X

good split $m \sim n/2$

bad split $m \ll n/2$ or $m \gg n/2$

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assume wolog that $m \ll n/2$

(length $Y = n - m - 1$)

$\underset{m}{[X]}$ pivot $\underset{n-m-1}{[Y]}$

if $m = \lceil n/2 \rceil - 1$ then pivot is <u>median</u>

if $m > \lceil n/2 \rceil - 1$ then median is in X

<u>and</u> its the $(m - \lceil n/2 \rceil + 1)^{th}$ largest element in X

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15 items

X: [1 10 0] pivot Y: [4]

↑ 3rd largest elt of x

So $10 - \lceil n/2 \rceil + 1$

$= 3$

$|X| = 10$

split based on pivot

X, p_1, Y,
4 . 5

QuickSelect (A, i) $n = |A|$

$1 \leq i \leq n$

(sorted $(A)[i]$)

p = pivot = $A[i]$

split into X < pivot

$Y \geq$ pivot

if $|X| = i-1$ then return pivot

else $|X| > i-1$ then

return $QS(X, i)$

else

return $QS(Y, i - |X| - 1)$ $=0$

worst case

$$T(n) = T(n-1) + \Theta(n)$$

$$= \sum_{i=1}^{n} \alpha i = \alpha\left(\frac{n(n-1)}{2}\right) = \Theta(n^2)$$

$T(n) = O(1) + O(n)$ ← best case

$T(n) = T(3n/4) + \Theta(n)$ ← If pivot is "good"

$n \quad\quad\quad\quad\quad\quad\quad\quad n$

$\downarrow \quad\quad\quad\quad\quad\quad\quad\quad 3/4\,n$

$3/4\,n$

$\downarrow \quad\quad\quad\quad\quad\quad\quad\quad 9/16\,n$

$9/16\,n$

$\downarrow$

$$\sum_{i=1}^{L} \left(3/4\right)^i n \quad \leq \quad 4n - \Theta(n)$$

- worst case QS $\Theta(n^2)$
- as long as pivot is "good"
  $\Theta(n)$
  $\equiv$ size of larger partition is $\leq c \cdot n$
  for $c < 1$
- if "approx median" is linear time
  then QS + app / med is $\Theta(n)$

Multiplication

374×225

$$
\begin{array}{r}
374 \\
225 \\
\hline
1870 \\
748 \\
748 \\
\hline
\end{array}
$$

5 × 374

2 × 374

$$
\begin{array}{c|ccc}
 & 2 & 2 & 5 \quad x \\
\hline
3 & & & \\
7 & & & \\
4 & & & \\
y & & &
\end{array}
$$

~~X[2]~~

$z = x \cdot y$

$Z[0] = X[0] \cdot y[0]$
(carrys)

$Z[1] = X[0] \cdot y[1] + y[0] \cdot X[1]$

$Z[2] = X[0]\,y[2] + X[1]\,y[1] + X[2]\,y[0]$

T(n) = # of single digit multiplications to compute product of 2 n-digit #'s

$= n^2$

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x   y   n - digits

$\underline{x^h \cdot 10^{n/2} + x^L} \qquad y^h \cdot 10^{n/2} + y^L$

n/2 dig #

$$x \cdot y = (x^h \cdot y^h) \cdot 10^n + (x^h y^L + y^h x^L)\, 10^{n/2} + (x^L \cdot y^L)$$

1234 · 5678

(12 · 100 + 34) (56 · 100 + 78)

= (12 · 56) · 100² + (12 · 78 + 34 · 56) · 100

$+ (78 \cdot 34)$

Split x, y into lower, upper halves
multiply all pairs (recursively)
Combine

$T(n) = 4T(n/2) + c$

$T(1) = 1$

$T(8) = 4T(4) = 16T(2) = 64T(1) = 64$

$T(16) = 4T(8) = 256$

$T(n) = \Theta(n^2)$