

## Today

- Complexity analysis
- Text decomposition
- DP

```
In [14]: def subset_sum(lst, target):
    print(f"{lst} {target}")
    if target < 0:
        return False
    if target == 0:
        return True
    if lst == []:
        return False
    return subset_sum(lst[1:], target-lst[0]) or \
           subset_sum(lst[1:], target)
```

Asymptotic Worst-case Runtime Complexity  
 Runtime of Alg is  $\Theta(f(n))$   
 $\uparrow$   
 size of the input

Complexity of multiplication

$\Theta(n^2)$	$\Theta(n^{\log_2 3})$	$\Theta(n \log n)$
Lattice	Karatsuba	Someone [2019]

$$1000 \times 2000$$

$$\sim (\log 1000)$$

$T(n) \rightarrow$  worst-case runtime of alg X on input of size  $n$

$$T(n) = \Theta(f(n))$$

$$T(n) \rightarrow O(f(n))$$

$$f(n) \text{ is } O(T(n))$$

Merge Sort is  $O(n^n)$   
is not  $\Theta(n^n)$

```
In [14]: def subset_sum(lst, target):
    print(f"{lst} {target}")
    if target < 0:
        return False
    if target == 0:
        return True
    if lst == []:
        return False
    return subset_sum(lst[1:], target-lst[0]) or \
        subset_sum(lst[1:], target)
```

$SS(n)$  - worst-case subset-sum  
on list of length  $n$

$$SS(n) \leq SS(n-1) + SS(n-1) + O(1)$$

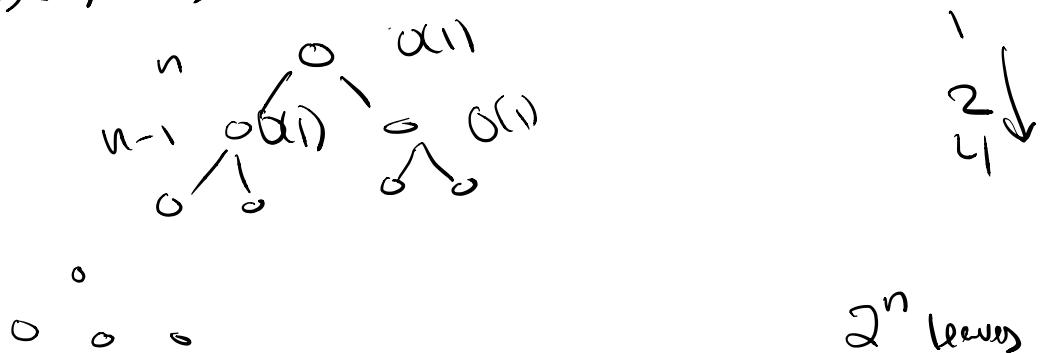
$$SS(n-1) \leq 2 SS(n-2) + O(1)$$

$$SS(0) = O(1)$$

$$SS'(n) = 2SS'(n-1) + O(1) \rightarrow \Theta(2^n)$$

$$SS'(0) = O(1)$$

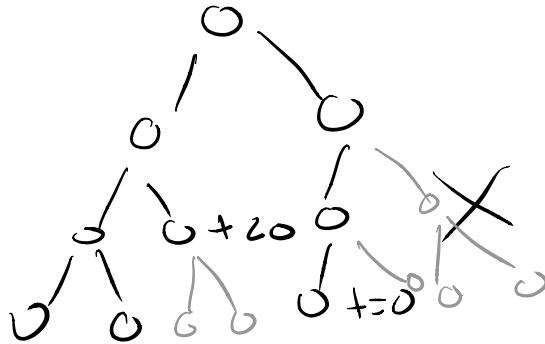
$$SS(n) \text{ is } O(SS'(n))$$



$2^n$  leaves

in a binary tree  
# of leaves is  $\Theta(2^n)$

$SS(n)$  is  $O(2^n)$



$$T(\underset{\substack{\text{subset-sum} \\ \text{root-level}}}{S}, \text{sum}(S)+1) = \Theta(2^n)$$

$$SS(n) \geq \underset{n}{\text{subset-sum}}(S, \text{sum}(S)+1)$$

$$SS(n) \overset{=} \Theta(2^n)$$

$$SS(n) \text{ is } \sum (2^n) [2^n \text{ is } O(SS(n))]$$

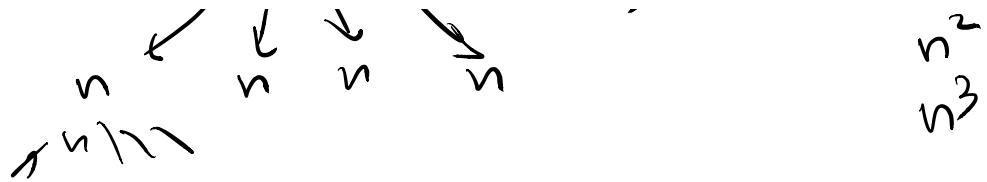
```

def nqueens(n, qsofar):
    col = len(qsofar)
    if col == n:
        return True
    count = 0
    for row in range(n):
        if queen_safe(row, qsofar):
            print(f"Placing new queen {qsofar} {row}")
            if nqueens(n, qsofar + [row]):
                return True
    return False
    
```

$NQ(n) \rightarrow$  runtime of  $nqueens(n, \{\})$

$\sum_n$

$n$   
/ / \ \ n calls



$O(n^n)$

Text segmentation

"this is a hard course" (no spaces)

this is a hard course |

this | is a hard course |

i | s a hard course | X

i s | a hard course |

a | hard course |

ha | rd course |

rd course | X

hard | course |

segment (string, n)

for i = 1 to len(string) - n:

if is word (string[n .. n+i])

if segment (string, n+i)

return True

return False