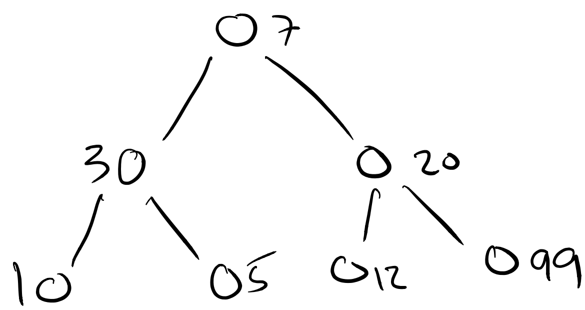
Today DP on trees

1. Binary search tree
2. Maximum independent set
3. [CYK]

Exam 2: 7-9 pm on Nov 5
- covers material up to & including next tue's lecture
- conflicts: LMK by **Tues**




```
bst_search(root, key)
    if root is None:
        return None
    if root.value == key:
        return root
    else if key < root.value:
        bst_search(root.left, key)
    else:
        bst_search(root.right, key)
```

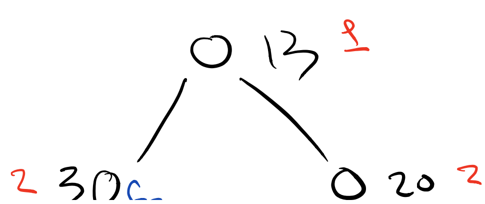
$\Theta(\log n)$

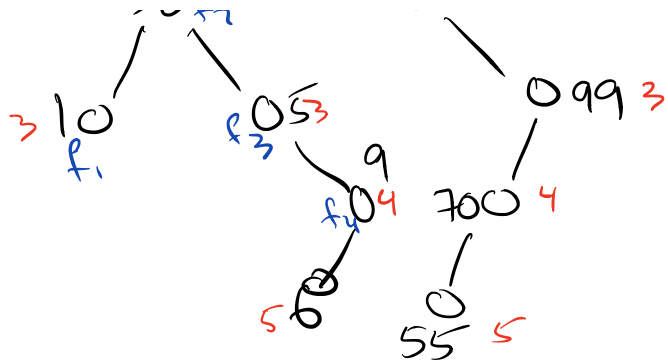
worst-case search time is h → height of tree

balanced tree h is $\Theta(\log n)$

# of elements in BST

unbalanced tree h is $\Theta(n)$ worst-case





level of element i

$$\frac{3+2+3+4+5+1+2+3+4+5}{10} = \frac{\sum h(i)}{n}$$

$$\sum f(i)\,\ell(i)$$

```
bst_ave_cost( root, level = 1):
    if root ~~is~~ None:
        return 0
    else:
        return level * root.f + bac(root.left, level+1)
                  ↑            + bac(root.right, level+1)
               cur node
```

Sorted list of values [1, 3, 5, 6, 9, 13, 20, 55, 70, 99]

list of frequencies $f_0, \ldots, f_9$

find ~~BST w/~~ least average search cost

output cost of least cost BST

$f_4 \cdot 1$

9

[1, 3, 5, 6]   [13, 20, 55, 70, 99]

```
OBST( fre_list, level)
    if list is empty
        return 0
    best = +∞
    for i = 0 to len(freq_list)-1:
        make i root ... level+1).
```

calculate cur = OBST(0..i-1 , level+1)
OBST(i+1 .. n , level+1)
+ f[i] · level
if cur < best:
best = cur
return best

OBST[i, j, ℓ] = cost of subtree containing lefts i..j at ~~level~~ ℓ

$$OBST[i,j,\ell] = 0 \quad \text{if} \quad i > j$$

$$OBST[i,j,\ell] = \min_{r \in i..j} \; OBST[i, r-1, \ell+1] \; OBST[r+1, j, \ell+1] + f[r] \cdot \ell$$

$$O(n^3)$$

~~#base case~~
~~for i=0 to n-1~~
~~j = i+1 to n-1~~
~~OBFS[i, j, ℓ+1] = +~~
~~l = n to 1~~ for

for i = 0 to n-1
for j = 0 to i
for l = 0 to n+1
OBFS[i, j, ℓ] = 0

for i = 0 to n-1
j = i+1 to max(n-1, i + (n - level)) :
OBFS[i, j, l] = min ...

$$OBFS[i,j,\ell] \quad \sum_{k=i}^{j} f(k) \cdot (\ell'(k) + (l-1))$$

$$OBFS[i,j,\ell+1] \quad \sum_{k=1}^{j} f(k) \, (\ell'(k) + \ell))$$

$$OBFS[i,j,\ell+1] = OBFS[i,j,\ell] + \sum_{k=1}^{j} f(k)$$