Today
- Optimal BST in $O(n^2)$
- Max ind set in trees
- CYK

Exam 2: Tue Nov 5 7–9pm
more info on website / piazza
conflict info due TODAY
Final: Tue Dec 17, 1:30–4:30pm
conflict info due FRIDAY

OBST[i, j, ℓ] = cost of subtree containing elts i..j at level ℓ

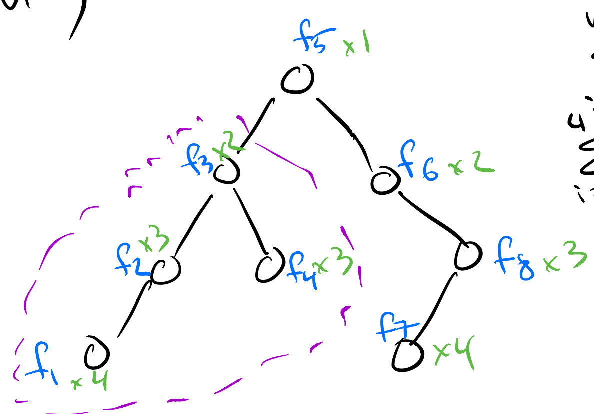
OBST[i, j, ℓ] = 0 if i > j

$$OBST[i,j,\ell] = \min_{r \in i..j} \; OBST[i, r-1, \ell+1] + OBST[r+1, j, \ell+1] + f[r]\cdot \ell$$

$O(n^3)$

$$\sum_{i=1}^{n} f[i]$$

$$\sum_{i=1}^{4} f[i] + \sum_{i=6}^{8} f[i]$$




$$OBST[i, j, \ell+1] = OBST[i, j, \ell] + \sum_{k=i}^{j} f[i]$$

$$OBST[i,j,\ell] = \min_{r \in i..j} \; OBST[i, r-1, \ell] + \sum_{k=i}^{r-1} f[k] + OBST[r+1, j, \ell]$$

$$+ \sum_{k=r+1}^{j} f[k]$$
$$+ f[r] \cdot \ell$$

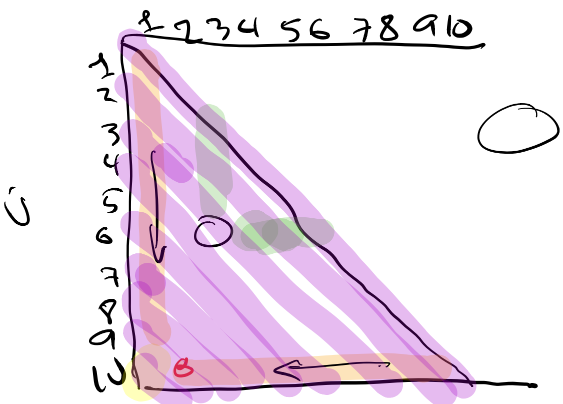
$$= \sum_{k=i}^{j} f[k] + \min \; OBST[i, r-1, \ell] + OBST[r+1, j, \ell] + f[r](\ell - 1)$$

OBST[1, n, 1]

OBST' [i, j] = cost of optimal subtree i..j rooted at level 1

OBST'[i, j] = 0 for i > j

$$OBST'[i,j] = \sum_{k=i}^{j} f[k] + \min_{r=i \text{ to } j} OBST'[i, r-1] + OBST'[r+1, j]$$




OBST'[1,10]

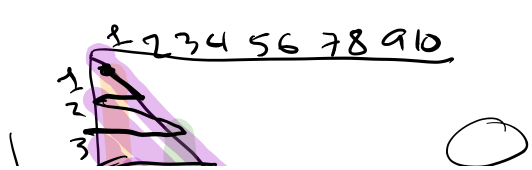
OBST [1,0]
OBST [2,10]

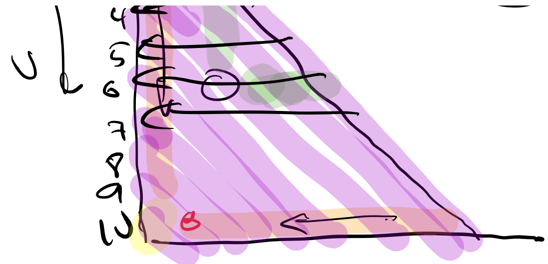
OBST [1, 1]
OBST [3, 10]

OBST[3,6]

OBST [4,6]
OBST [3,3]
OBST [5,6]
OBST [3,4]
OBST [6,6]

for d = 0 to 9
  for i = 1 to n − d
    OBST [i, i+d] = ⌴



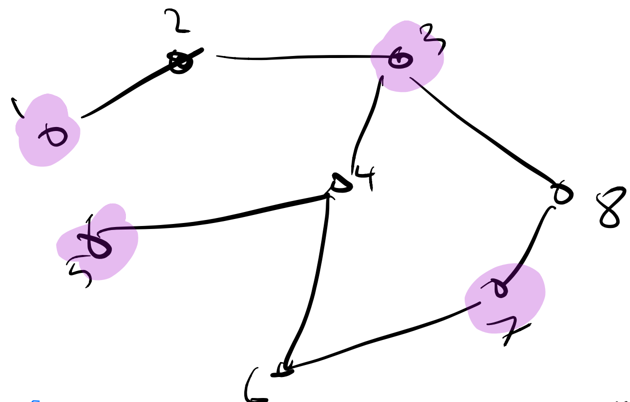


# Maximum Independent Set

$G = (V, E)$ $\qquad E \in V^2$

edge $(v_1, v_2)$ (directed)

$\{v_1, v_2\}$ undirected



independent set

$S \subseteq V$

s.t. $\forall x, y \in S$

$\{x, y\} \notin E$

<u>max</u> ind set

MIS[1..8] → MIS{1,2,4,5,6}

adjacency lists

1: [2]

2: [1, 3]

3: [2, 4, 8]

4:

adjacency matrix

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | | 1 | | 1 | | | | |
| 4 | | | 1 | | | 1 | | |
| 5 | | | | | | | | |
| 6 | | | | | | | | |
| 7 | | | | | | | | |
| 8 | | | 1 | | | | 1 | |

array of [linked] lists

is $v_1$ adjacent to $v_2$ $\qquad O(\deg(v_1))$ $\qquad O(V)$

$O(\log \deg(v_1))$ $\qquad O(\log V)$

space $O(E + V)$

<u>matrix</u>

is $v_1$ adj to $v_2$ $O(1)$

enumerate all edges $O(V^2)$

space $O(V^2)$

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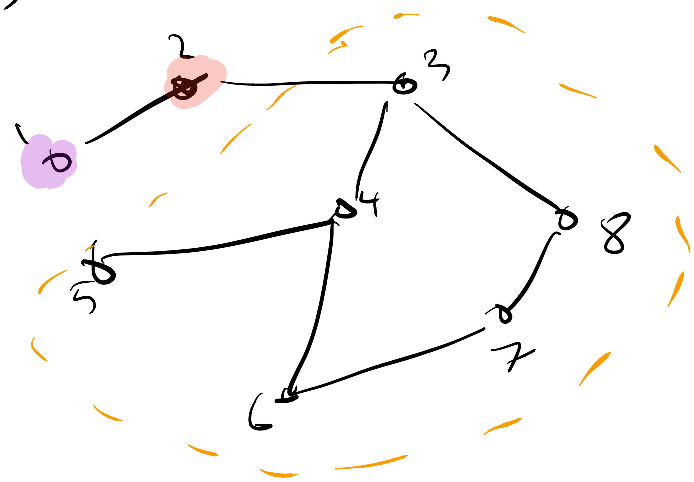
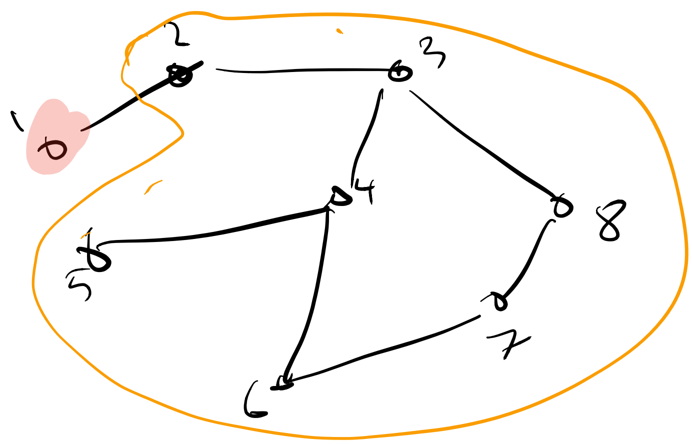
MIS




MIS ( G )

pick a node in G → v

Consider MIS includes v

$1 + MIS(G - \{v\} - \{N(v)\})$

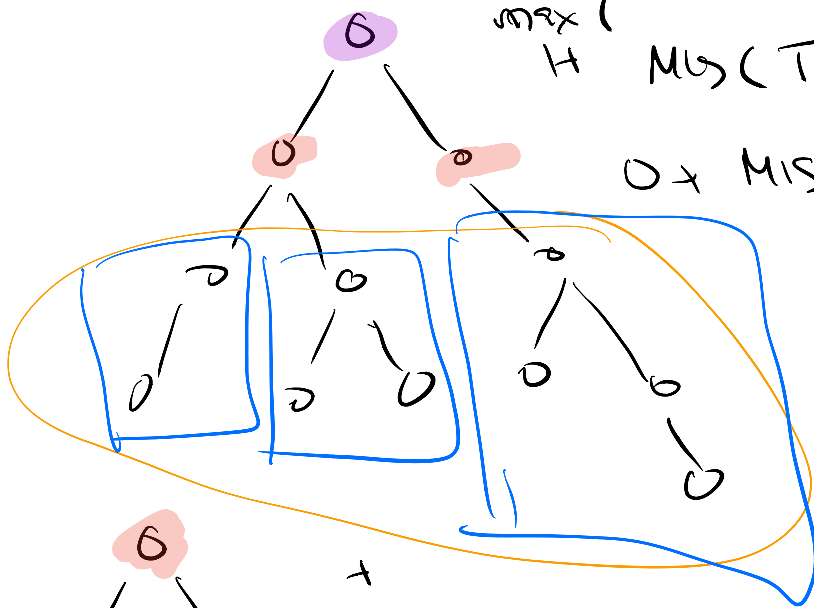
consider MIS w/o v

$0 + MIS(G - \{v\})$

MIS on graphs → NP-complete conjectured to require >

MIS on trees

"poly" time



$$\max\left( v + MIS(T - \{v\} - \{N(v)\}),\ 0 + MIS(T - \{v\}) \right)$$

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