TODAY
- finish Gale Shapley
- NP-hardness
  - Decision problems
  - P & NP
  - NP hardness
  - reductions

Gale Shapley

Each round:

1. Find unmatched company
2. Make offer to highest ranked student previously not offered
3. student accepts unless has better offer (possibly rejecting current offer)

Lemma 1 G-S terminates

Proof Co makes an offer to a student only once

at most n^2 offers

Lemma 2 G-S finds a perfect matching

Proof suppose s is unmatched
then s got no offers
suppose C is unmatched
then C made an offer to all students
∴ at end of G-S everyone is matched.

Lemma 3 G-S finds stable matching

Proof s_1 matched with C_1 but prefers C_2
C_2 matched w/ s_2 but prefers s_1

C_2 must have made offer to s_1 and been rejected

C_1 must be preferred by s_1 to C_2

Efficient → polynomial
$O(n^c)$ for c constant

(real-world efficiency
not sufficient cond n^{1000}
not entirely necessary)

Decision Problem

$f: X \to \{0,1\}$
true/false

Shortest path

find [length] of shortest path from s to t

DP is there a path from s to t w/ length $\leq \ell$

MIS find size of MIS

DP is there a IS $\geq k$

has-is $(G, k) \to$ true/false

has-is $(G, 1000) \to F$

has-is $(G, 1) \to T$

for $i = 1$ to $|V|$

has_ is (G, i)
max

P = set of decision problems w/ $O(n^c)$ algs

NP = set of decision problems with efficiently checkable certificates for "YES"

Certificate: Given a DP f and some instance X, c is a certificate if $\exists$ $v : X, c \to T/F$ such that $\exists\, c$ with $v(X, c) = T$ iff $f(X) = T$

DP: is there a path from $s \to t$ w/ length $\leq \ell$

DP: is there an IS with at least k vertices

DP: is there a subset of S that sums to ϵ?

$P \subseteq NP$ $\quad P \stackrel{?}{=} NP$

NP-hard: DP X is NP-hard

- when if $X \in Y$ then $Y = NT$