Proving L is regular
- show DFA, NFA, regex
- else closure properties

2A: $L = \{ \min\{ \#0(x), \#1(x) \} \geq 4 \}$

(i) $L_1 = \{ \#0(x) \geq 4 \}$
$L_2 = \{ \#1(x) \geq 4 \}$

$L_1 = 1^* 0 1^* 0 1^* 0 1^* 0 (1+0)^*$

$L = L_1 \cap L_2$

$0000 \in L_1$
$\notin L_2$
$\notin L$

(ii) ~~$L^c = \{ \#0(x) < 4$ and $\#1(x) < 4 \}$~~

~~$x \in L^c$ $|x| \leq 6$~~

$\cancel{(1^*(0+\epsilon)1^*(0+\epsilon)1^*(0+\epsilon)1^*} +$
$0^*(1+\epsilon)0^*(1+\epsilon)0^*(1+\epsilon)0^*$

2B $L = \{ x \in \{0,1\}^* \mid \min(\#(0,x), \#(1,x))$ divisible by 5 $\}$

0^{5n}

0^{5i} 0^{5j} $i \neq j$

~~$F \subseteq L$~~

$i < j$ (wolog)

1^{5i+1}

$5i < 5i+1 < 5j$

$0^{5i} 1^{5i+1} \in L$

$0^{5j} 1^{5i+1} \notin L$

$$\frac{0^{5i}\ s}{\in L} \qquad \frac{0^{5j}\ s}{\notin L}$$

① #0 div 5
#1 not div 5

② $s = 1$
$\#0 \leq \#1$
$\#0 > \#1$
j

$F = 0^n$

$0^i \quad 0^j$

$i \neq j \mod 5$

Find regex for $L = \{0^n w 1^n \mid n \geq 1 \wedge w \in \Sigma^+\}$

$0^n \underbrace{\quad}_{w} 1^n$

$0 \underbrace{0^{n-1} \underbrace{w} 1^{n-1}}_{w'} 1$

$\underline{0}(0\,011\,0101011\,0)\underline{1}$

$0(0+1)^+1$

$x \in L \qquad x' \notin L$

$\{x, x'\}$ is a fooling set

maximum size fooling set is finite

$L \subset \{0\}^*$

L^*

$skip(x)$ = odd pos symbols in x

$skip(NIKITA) = NKT$

$skip(Algorithm) = Agrtm$

L is regular $\Rightarrow$ $L' = \{skip(x) \mid x \in L\}$ regular

$\exists M = (\Sigma, Q, \delta, s, A)$ (dfa)

$L(M) = L$

$N = (\Sigma, Q, \delta', s, A)$

$\delta'(q, a) =$ all states that M can reach from q on input a[?]

$= \{ \delta(\delta(q,a),c) \mid c \in \Sigma \}$

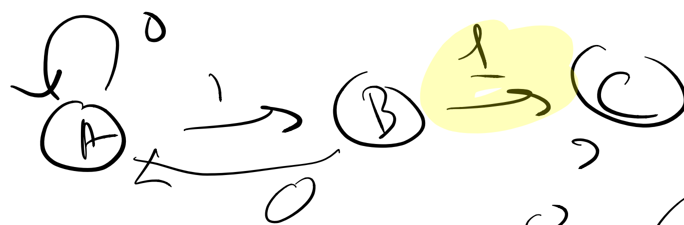
$Q' = Q \times \{0,1\}$

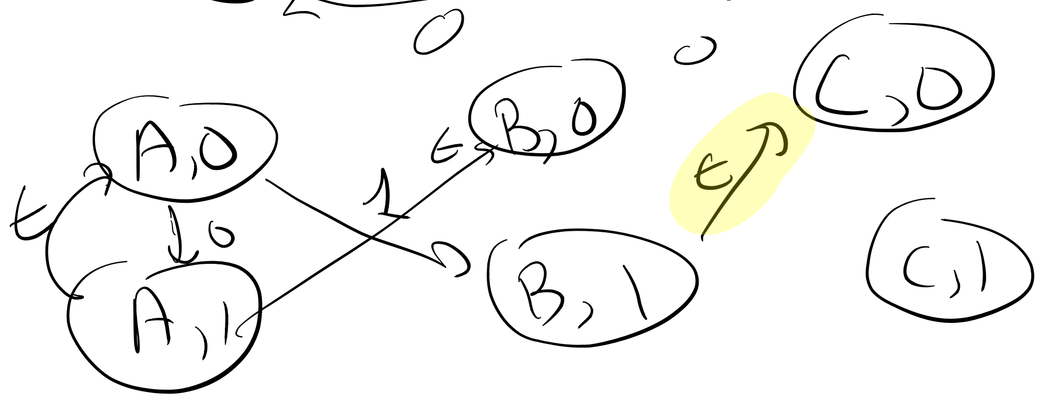
$s' = (s,0)$

$\delta'((q,0),a) = (\delta(q,a),1)$

$\delta'((q,1),\epsilon) = \{(\delta(q,c),0) \mid c \in \Sigma\}$

DFA





01011

A A B A B C

0 ε 0 ε 1

A,0 A,1 A,0 B,1 A,0 B,1 C,0

For every CFL L,

$L' = \{ 0^{|w|} \mid w \in L \}$ is CFree

For regular language L

is $L' = \{0^{|w|} \mid w \in L\}$ reg?

r is regex L

$\emptyset, \epsilon, a \in \Sigma$

transform(r) = replacing $a \in \Sigma$ with 0

$(0-9)^* (374) (0-9)^* +$

$(0-9)^* (473) (0-9)^*$

$(0+1+2+3+ \ldots + 9)^* (374)$

$(0+0+0+ \ldots +0)^* (000)$

$S = SS \mid (S) \mid \epsilon$

$S = SS \mid \emptyset S \emptyset \mid \epsilon$

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x, y are equivalent if

$\forall z \quad xz \in L \iff yz \in L$

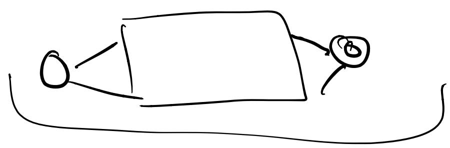
$x \equiv_L y$

Fooling set F each $x, y \in F$ $x \neq y$

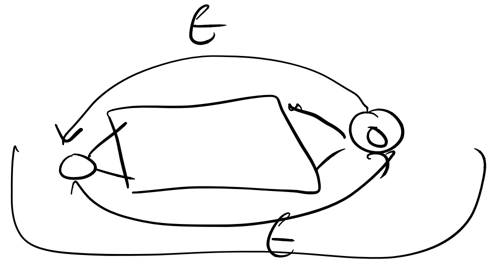
$x \not\equiv y$

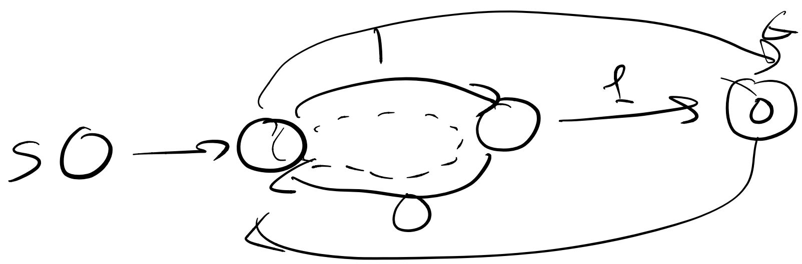
x, y are in different equivalence classes

r_1

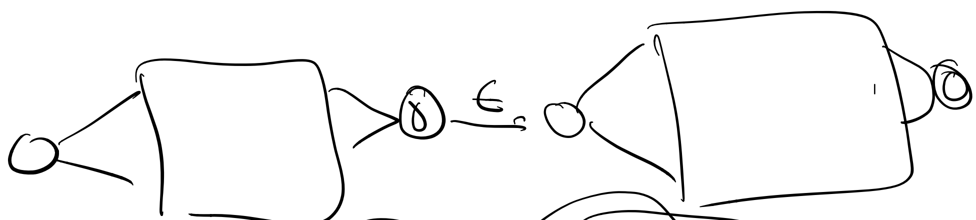


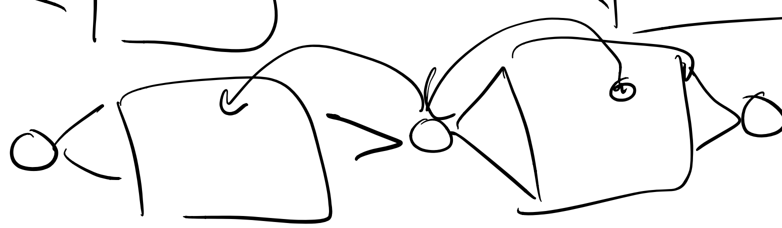
r_1^*





$((ab)^* c)^*$ ϵ $\to ((ab^*)(\epsilon + c))^*$





$\delta'(q, a) = \delta(q, a) \cap Q'$

$x \in L(M')$ $\qquad \exists a \in A'$

$a \in \delta'^*(s, x)$

$q_1, \dots, q_n$ $\qquad s = q_1, \quad a = q_n$

$q_{i+1} \in \delta'(q_i, x[i])$ $\quad q_{i+1} \in \delta(q_i, \cancel{x[i]})$

$\Rightarrow q_{i+1} \in \delta(q_i, x[i])$

$y = x$ $y[i]$ ~~with~~ ϵ inserted

$\rightarrow q \in \delta^*(s, x)$.

$L = \{0^n 1^{an+b}\}$ for some $a, b \geq 0$

$\boxed{L = \{0^n 1^{5n+7}\}}$

$L' = \{0^n 1^{an+b} \mid n, a, b \geq 0\}$

$S \rightarrow A 1^b$

$A \rightarrow 0 A 1^a \mid e$

$L = \{0^n 1^{an-b} \mid n \geq b/a\}$

$L' = \{0^n 1^{5n-7}\}$

$L'' = \{0^n 1^{5(n-7)}\}$

$= \{0^{n+7} 1^{5n}\}$

$L' = \{0^n 1^{5n-7}\} = \{0^n 1^{5(n-2)} 1^3\}$

$= \{0^{i} 0^{..} 1^{5..} 1^{5j}\}$

$0^{i} 1^{j} 2^{k}$ $\qquad$ $i - j < k$

$1 < k + j$

$0^{n} 1^{n}$ $\qquad$ $1 \to 11111$

$0^{n} 1^{5n}$

L if $\{w^{R} \mid w \in L\}$ is not regular then L is not regular

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if L is regular then $w^{R} \mid w \in L$ is regular

$L_1 \quad L_2 \quad L_3$

$L = L_1 + L_2 L_3^{*}$

$L_1 = \; S_1 \to A \mid B \dots$

$L_2 = S_2 \to X \mid Y \dots$

$L_1 + L_2$

$S \to S_1 \mid S_2$

$L_1 \circ L_2$

$S \to S_1 S_2$

L_1^*

$S \to A$

$A \to S_1 A \mid \epsilon$

$S \to S_1 S \mid \epsilon$

$S \to S_1 \mid S_2 A$ $\quad L = L_1 + L_2 L_3^*$

$A \to S_3 A \mid \epsilon$

$G_i = (V_i, T, P_i, S_i)$

$$G = (V_1 \cup V_2 \cup V_3 \cup \{S, A\},\ T,\ P_1 \cup P_2 \cup P_3 \cup \{S \to S_1 \mid S_2 A,\ A \to S_3 A \mid \epsilon\},\ S)$$

$$\text{stutter}(w) = \begin{cases} \epsilon & \text{if } w = \epsilon \\ aa\ \text{stutter}(x) & \text{if } w = ax \end{cases}$$

$374 \to 337744$

(d) w is a palindrome iff stutter(w) is a palindrome

w is a palindrome

$w = axa$ for $a \in \Sigma$, $x = x^R$

$= a$

$= \epsilon$

$$\begin{aligned} \text{stutter}(w) &= \text{stutter}(axa) \\ &= aa(\text{stutter}(xa)) \\ &= aa\ \overline{\text{stutter}(x)\ aa} \\ &\quad (aa\ \ s(x)\ aa)^R \\ &= aa\ \ s(x)^R\ aa \end{aligned}$$

$s(x)^R = s(x)$ since $x = x^R$ IH