HW 2

Due on **Wednesday, September 9, 2020 at 10am CST**

CS/ECE 374: Algorithms & Models of Computation, Fall 2020 Version: **1.3**

Submission instructions as in previous homeworks.

4 (100 PTS.) Regular expressions.

For each of the following languages over the alphabet $\{0, 1\}$, give a regular expression that describes that language, and briefly argue why your expression is correct.

4.A. (10 PTS.) All strings that end in 1011.

4.B. (10 PTS.) All strings except 11.

4.C. (10 PTS.) All strings that contain 101 or 010 as a substring.

4.D. (10 PTS.) All strings that contain 111 and 000 as a subsequence (the resulting expression is long – describe how you got your expression, instead of writing it out explicitly).

4.E. (10 PTS.) The language containing all strings that do not contain 111 as a substring.

4.F. (10 PTS.) All strings that do *not* contain 000 as a subsequence.

4.G. (10 PTS.) Strings in which every occurrence of the substring 00 appears before every occurrence of the substring 11.

4.H. (10 PTS.) Strings that do not contain the subsequence 010.

4.I. (10 PTS.) Strings that do not contain the subsequence 0101010.

4.J. (10 PTS.) Strings that do not contain the subsequence 10.

4.K. (Not for credit, do not submit a solution.) Strings that do not contain the subsequence 111000.

5 (100 PTS.) DFA I

Let $\Sigma = \{0, 1\}$. Let L be the set of all strings in Σ^* that contain an even number of 0s and an even number of 1s.

5.A. (50 PTS.) Describe a DFA over Σ that accepts the language L. Argue that your machine accepts every string in L and nothing else, by explaining what each state in your DFA *means*. (Hint: Zero is even)

You may either draw the DFA or describe it formally, but the states Q, the start state s, the accepting states A, and the transition function δ must be clearly specified, in either case.

5.B. (50 PTS.) (Harder.) Give a regular expression for L, and briefly argue why the expression is correct. (Hint: First solve the much easier case where the strings do not contain any consecutive 0s or 1s.)

6 (100 PTS.) DFA II

Let L_1, L_2, and L_3 be regular languages over Σ accepted by DFAs $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$, $M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$, and $M_3 = (Q_3, \Sigma, \delta_3, s_3, A_3)$, respectively.

6.A. (20 PTS.) Describe formally the product construction of the DFA M that accepts the language $L_1 \cap L_2 \cap L_3$.

6.B. (30 PTS.) In the DFA M constructed in (**??**), a state is a triple (q_1, q_2, q_3). Let δ the transition function of M, and let δ^* be the standard extension of δ to strings. Prove by induction that for any string $w \in \Sigma^*$, we have that

$$\delta^*\big((q_1, q_2, q_3), w\big) = \big(\delta_1^*(q_1, w), \delta_2^*(q_2, w), \delta_3^*(q_3, w)\big).$$

6.C. (20 PTS.) Describe a DFA $M = (Q, \Sigma, \delta, s, A)$ in terms of M_1, M_2, and M_3 that accepts $L = \left\{ w \;\middle|\; w \text{ is in exactly two of } \{L_1, L_2, L_3\} \right\}$. Formally specify the components Q, δ, s, and A for M in terms of the components of M_1, M_2, and M_3. Argue that your construction is correct.

6.D. (30 PTS.) You are given a DFA $M = (Q, \Sigma, \delta, s, A)$, for $\Sigma = \{0, 1\}$. Describe in detail how to build a DFA that accepts the language

$$L = \left\{ w \in \Sigma^* \;\middle|\; w \notin L(M), \overline{w} \in L(M) \text{ and } 1^{|w|} \in L(M) \right\}.$$

How many states does your DFA has as a function of $n = |Q|$? Argue that the DFA you constructed indeed accepts the specified language.

Here, for $w = w_1 w_2 \dots w_m \in \Sigma^*$, the *complement string* $\overline{w}$ is $\overline{w_1}\,\overline{w_2}\,\overline{w_3} \dots \overline{w_m}$, where $\overline{0} = 1$, and $\overline{1} = 0$.