4.1.1

Formal definition of NFA

Reminder: Power set

Q: a set. Power set of Q is: $\mathcal{P}(Q) = 2^Q = \{X \mid X \subseteq Q\}$ is set of all subsets of Q.

Example

$Q = \{1, 2, 3, 4\}$

$$\mathcal{P}(Q) = \left\{ \begin{array}{c} \{1,2,3,4\}, \\ \{2,3,4\}, \{1,3,4\}, \{1,2,4\}, \{1,2,3\}, \\ \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \\ \{1\}, \{2\}, \{3\}, \{4\}, \\ \{\} \end{array} \right\}$$

Formal Tuple Notation

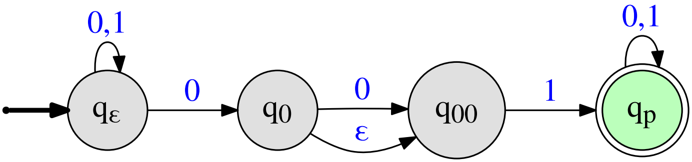
Definition

A non-deterministic finite automata (NFA) $N = (Q, \Sigma, \delta, s, A)$ is a five tuple where

- Q is a finite set whose elements are called states,
- Σ is a finite set called the input alphabet,
- $\delta : Q \times \Sigma \cup \{\varepsilon\} \rightarrow \mathcal{P}(Q)$ is the transition function (here $\mathcal{P}(Q)$ is the power set of Q),
- $s \in Q$ is the start state,
- $A \subseteq Q$ is the set of accepting/final states.

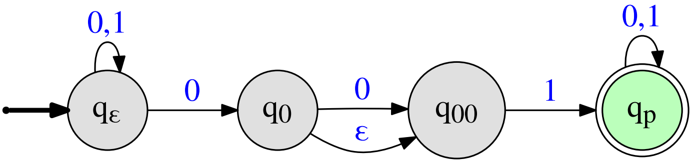
$\delta(q, a)$ for $a \in \Sigma \cup \{\varepsilon\}$ is a subset of Q — a set of states.

Example



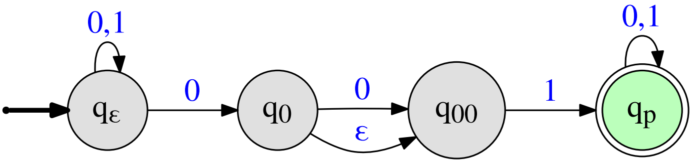
- $Q = \{q_\varepsilon, q_0, q_{00}, q_p\}$
- $\Sigma = \{0, 1\}$
- δ
- $s = q_\varepsilon$
- $A = \{q_p\}$

Example



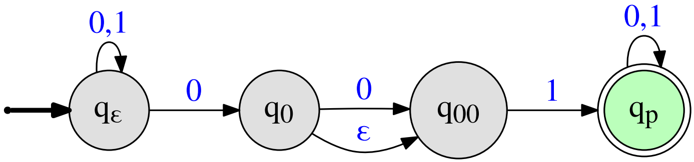
- $Q = \{q_\varepsilon, q_0, q_{00}, q_p\}$
- $\Sigma = \{0, 1\}$
- δ
- $s = q_\varepsilon$
- $A = \{q_p\}$

Example



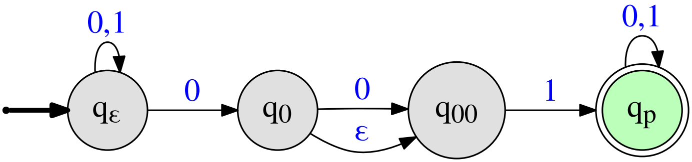
- $Q = \{q_\varepsilon, q_0, q_{00}, q_p\}$
- $\Sigma = \{0, 1\}$
- δ
- $s = q_\varepsilon$
- $A = \{q_p\}$

Example



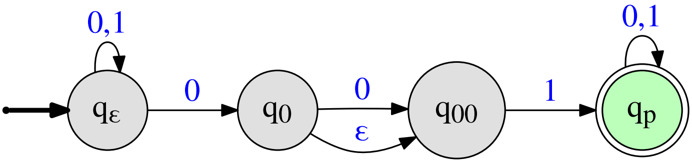
- $Q = \{q_\varepsilon, q_0, q_{00}, q_p\}$
- $\Sigma = \{0, 1\}$
- δ
- $s = q_\varepsilon$
- $A = \{q_p\}$

Example



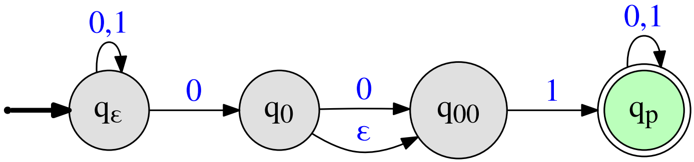
- $Q = \{q_\varepsilon, q_0, q_{00}, q_p\}$
- $\Sigma = \{0, 1\}$
- δ
- $s = q_\varepsilon$
- $A = \{q_p\}$

Example



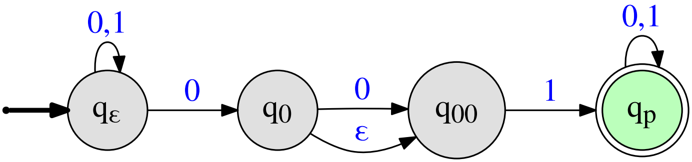
- $Q = \{q_\varepsilon, q_0, q_{00}, q_p\}$
- $\Sigma = \{0, 1\}$
- δ
- $s = q_\varepsilon$
- $A = \{q_p\}$

Example



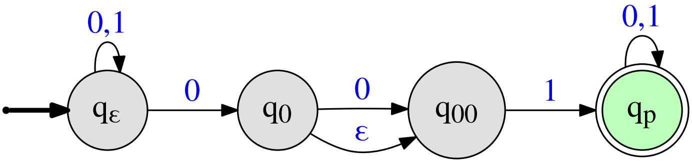
- $Q = \{q_\varepsilon, q_0, q_{00}, q_p\}$
- $\Sigma = \{0, 1\}$
- δ
- $s = q_\varepsilon$
- $A = \{q_p\}$

Example



- $Q = \{q_\varepsilon, q_0, q_{00}, q_p\}$
- $\Sigma = \{0, 1\}$
- δ
- $s = q_\varepsilon$
- $A = \{q_p\}$

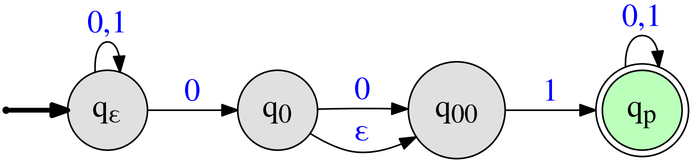
Example



- $Q = \{q_\varepsilon, q_0, q_{00}, q_p\}$
- $\Sigma = \{0, 1\}$
- δ
- $s = q_\varepsilon$
- $A = \{q_p\}$

Example

Transition function in detail...



$$\delta(q_\varepsilon, \varepsilon) = \{q_\varepsilon\}$$
$$\delta(q_\varepsilon, 0) = \{q_\varepsilon, q_0\}$$
$$\delta(q_\varepsilon, 1) = \{q_\varepsilon\}$$

$$\delta(q_0, \varepsilon) = \{q_0, q_{00}\}$$
$$\delta(q_0, 0) = \{q_{00}\}$$
$$\delta(q_0, 1) = \{\}$$

$$\delta(q_{00}, \varepsilon) = \{q_{00}\}$$
$$\delta(q_{00}, 0) = \{\}$$
$$\delta(q_{00}, 1) = \{q_p\}$$

$$\delta(q_p, \varepsilon) = \{q_p\}$$
$$\delta(q_p, 0) = \{q_p\}$$
$$\delta(q_p, 1) = \{q_p\}$$

THE END

...

(for now)