Graph Search

Lecture 15

Thursday, October 15, 2020

LaTeXed: September 16, 2020 09:16

15.1
Graph Basics

Why Graphs?

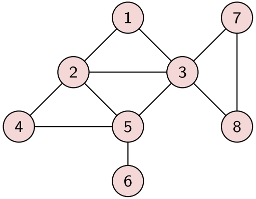
1. Graphs help model networks which are ubiquitous: transportation networks (rail, roads, airways), social networks (interpersonal relationships), information networks (web page links), and many problems that don't even look like graph problems.
2. Fundamental objects in Computer Science, Optimization, Combinatorics
3. Many important and useful optimization problems are graph problems
4. Graph theory: elegant, fun and deep mathematics

Graph

Definition

An undirected (simple) graph $G = (V, E)$ is a 2-tuple:

1. V is a set of vertices (also referred to as nodes/points)
2. E is a set of edges where each edge $e \in E$ is a set of the form $\{u, v\}$ with $u, v \in V$ and $u \neq v$.



Example

In figure, $G = (V, E)$ where $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $E = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 5\}, \{3, 7\}, \{3, 8\}, \{4, 5\}, \{5, 6\}, \{7, 8\}\}$.

Example: Modeling Problems as Search

State Space Search

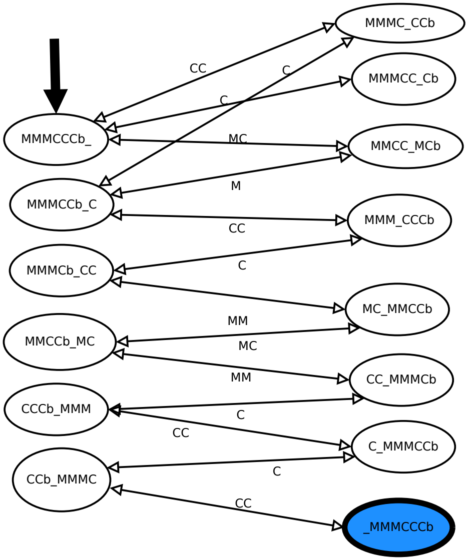
Many search problems can be modeled as search on a graph.
The trick is figuring out what the vertices and edges are.

Missionaries and Cannibals

- Three missionaries, three cannibals, one boat, one river
- Boat carries two people, must have at least one person
- Must all get across
- At no time can cannibals outnumber missionaries

How is this a graph search problem?
What are the vertices?
What are the edges?

Cannibals and Missionaries: Is the language empty?



Problems goes back to 800 CE
Versions with brothers and sisters.
Jealous Husbands.
All bad names to a simple problem...

Problems on DFAs and NFAs sometimes are just problems on graphs

1. M: DFA/NFA is $L(M)$ empty?
2. M: DFA is $L(M) = \Sigma^*$?
3. M: DFA, and a string w. Does M accepts w?
4. N: NFA, and a string w. Does N accepts w?

Exercise

State the following problems as graph problems, and describe an algorithm that solves them (we will solve them later on in the course):

1. M: DFA, is $L(M)$ infinite?
2. N: NFA, is $L(M)$ finite?
3. M: DFA/NFA, compute the shortest word in $L(M)$?
4. M: DFA, if $L(M)$ is finite, compute the longest word $w \in L(M)$?

[Solutions would probably not be recorded for these questions (lack of time).]

THE END

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(for now)