16.4

DFS in Directed Graphs

DFS

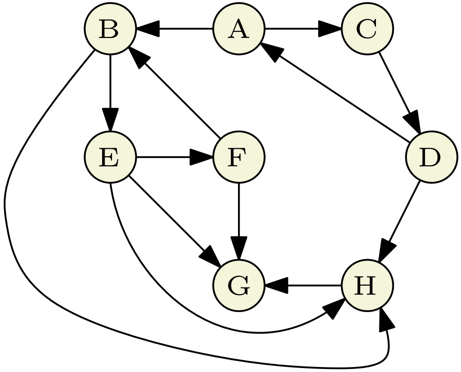
16.4.1
DFS in Directed Graphs: Pre/Post numbering

DFS in Directed Graphs

```
DFS(G)
    Mark all nodes u as unvisited
    T is set to ∅
    time = 0
    while there is an unvisited node u do
        DFS(u)
    Output T
```

```
DFS(u)
    Mark u as visited
    pre(u) = ++time
    for each edge (u, v) in Out(u) do
        if v is not visited
            add edge (u, v) to T
            DFS(v)
    post(u) = ++time
```

Example of DFS in directed graph



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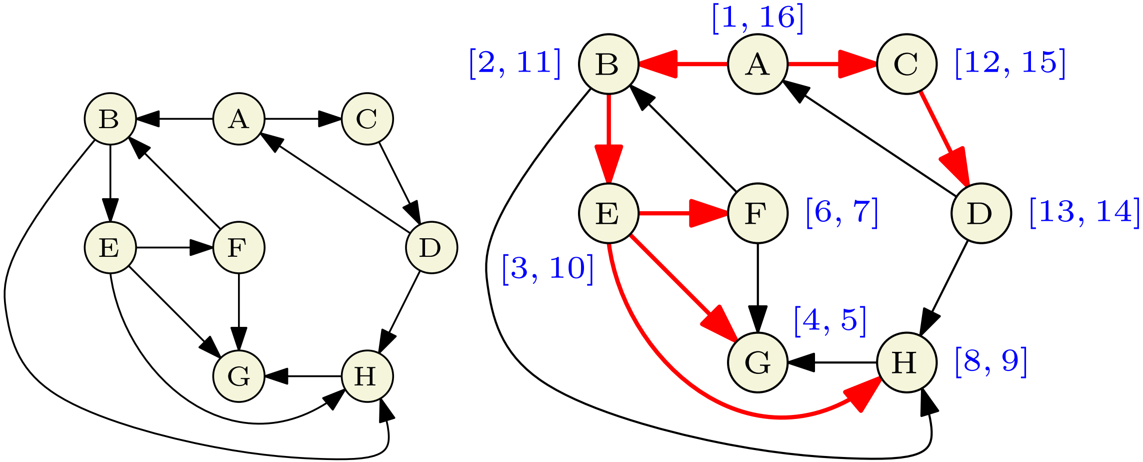

DFS Properties

Generalizing ideas from undirected graphs:

1. **DFS**(G) takes $O(m+n)$ time.
2. Edges added form a branching: a forest of out-trees. Output of $DFS(G)$ depends on the order in which vertices are considered.
3. If u is the first vertex considered by $DFS(G)$ then $DFS(u)$ outputs a directed out-tree T rooted at u and a vertex v is in T if and only if $v \in \mathrm{rch}(u)$
4. For any two vertices x, y the intervals $[\mathbf{pre}(x), \mathbf{post}(x)]$ and $[\mathbf{pre}(y), \mathbf{post}(y)]$ are either disjoint or one is contained in the other.

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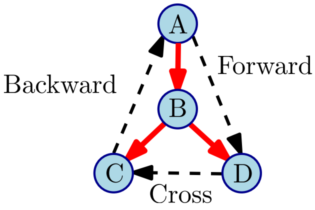
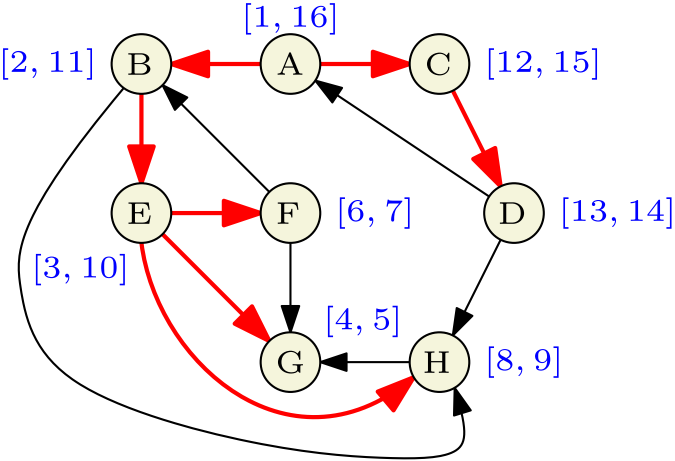
DFS tree and related edges

Edges of G can be classified with respect to the **DFS** tree T as:

1. **Tree edges** that belong to T
2. A **forward edge** is a non-tree edges (x, y) such that $\text{pre}(x) < \text{pre}(y) < \text{post}(y) < \text{post}(x)$.
3. A **backward edge** is a non-tree edge (y, x) such that $\text{pre}(x) < \text{pre}(y) < \text{post}(y) < \text{post}(x)$.
4. A **cross edge** is a non-tree edges (x, y) such that the intervals $[\text{pre}(x), \text{post}(x)]$ and $[\text{pre}(y), \text{post}(y)]$ are disjoint.



Types of Edges



THE END

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(for now)