17.2
Breadth First Search

Breadth First Search (BFS)

Overview

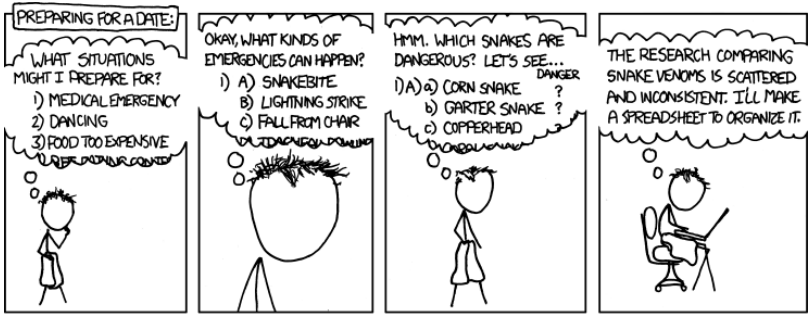
A. **BFS** is obtained from **BasicSearch** by processing edges using a **queue** data structure.

B. It processes the vertices in the graph in the order of their shortest distance from the vertex ***s*** (the start vertex).

As such...

1. **DFS** good for exploring graph structure
2. **BFS** good for exploring distances

xkcd take on DFS



Queue Data Structure

Queues

A **queue** is a list of elements which supports the operations:

1. **enqueue**: Adds an element to the end of the list
2. **dequeue**: Removes an element from the front of the list

Elements are extracted in **first-in first-out (FIFO)** order, i.e., elements are picked in the order in which they were inserted.

BFS Algorithm

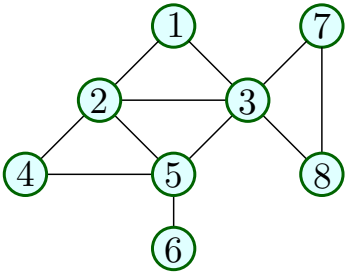
Given (undirected or directed) graph $G = (V, E)$ and node $s \in V$

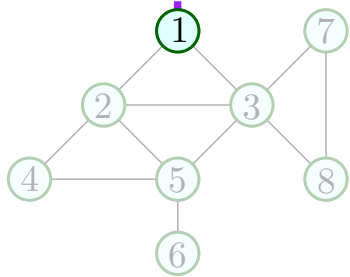
```
BFS(s)
    Mark all vertices as unvisited
    Initialize search tree T to be empty
    Mark vertex s as visited
    set Q to be the empty queue
    enqueue(Q, s)
    while Q is nonempty do
        u = dequeue(Q)
        for each vertex v ∈ Adj(u)
            if v is not visited then
                add edge (u, v) to T
                Mark v as visited and enqueue(v)
```

Proposition

BFS(s) *runs in* $O(n + m)$ *time.*

BFS: An Example in Undirected Graphs





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T9. []

BFS tree is the set of purple edges.

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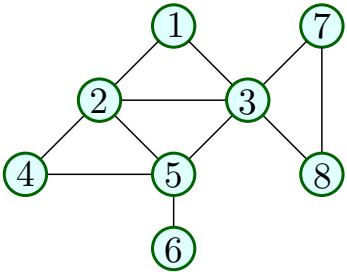

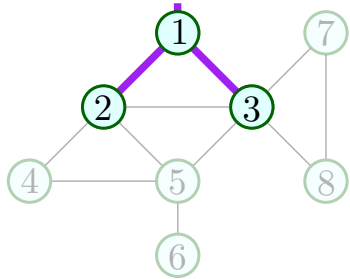

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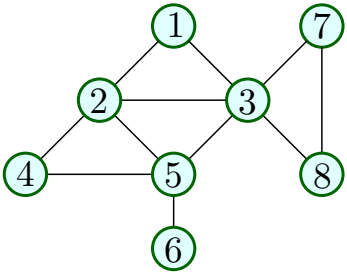

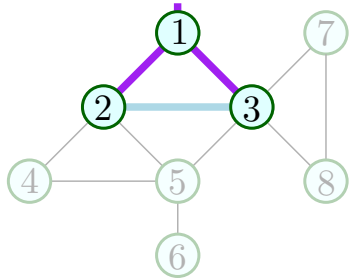

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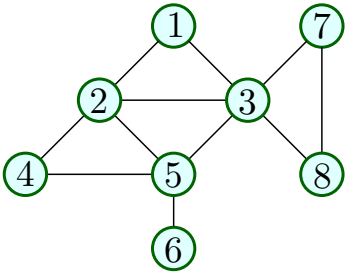

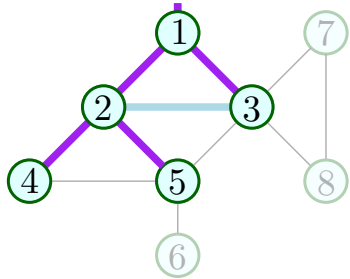

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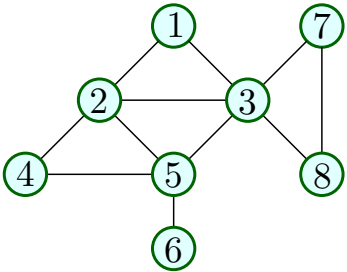

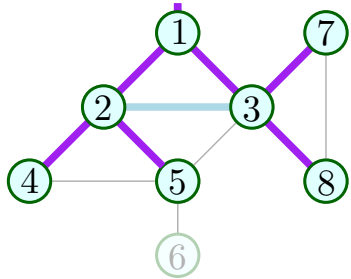

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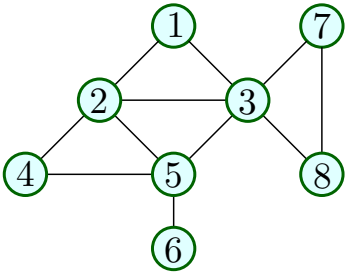

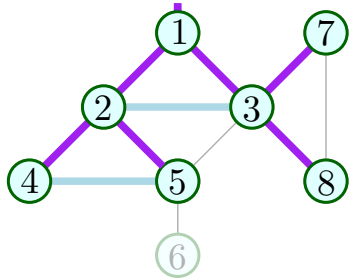

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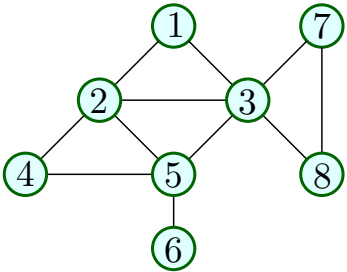

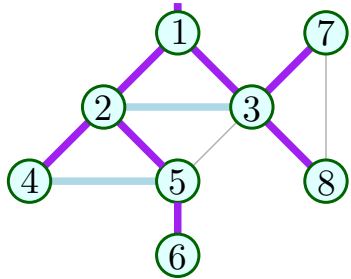

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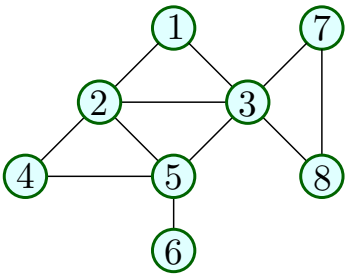

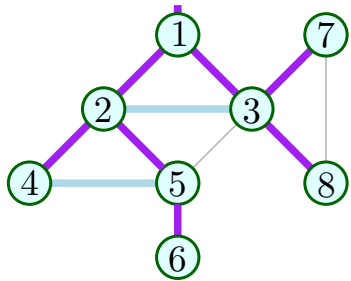

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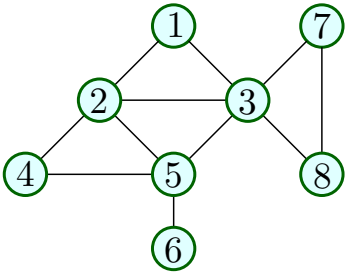

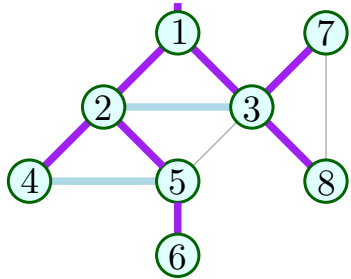

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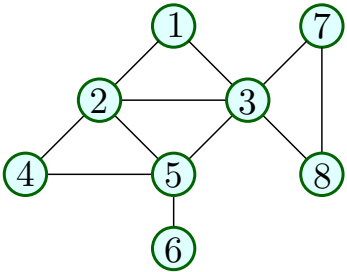

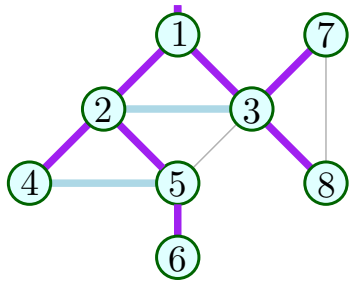

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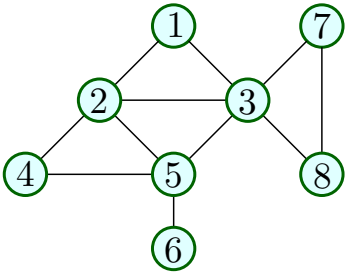

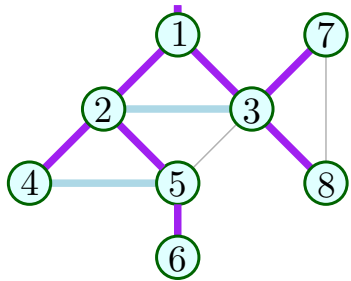

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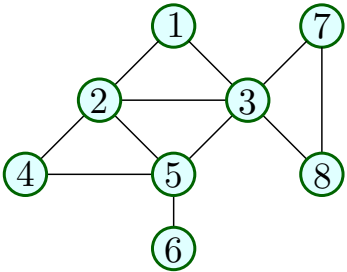

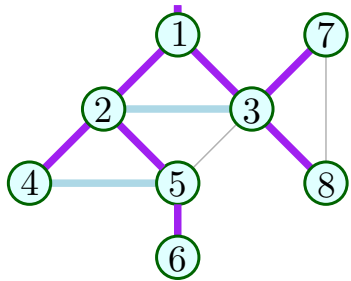

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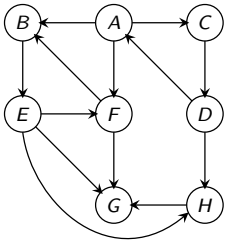
BFS: An Example in Directed Graphs



THE END

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(for now)