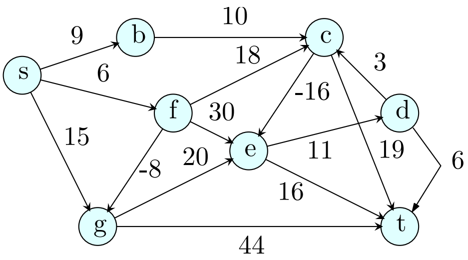
18.1.2
But wait! Things get worse: Negative cycles

Negative Length Cycles

Definition 18.2.

A cycle C is a negative length cycle if the sum of the edge lengths of C is negative.



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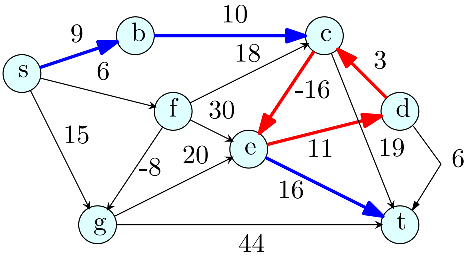

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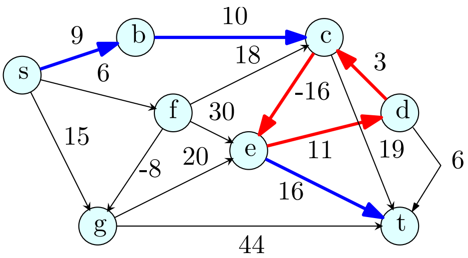

What is the shortest path distance between s and t?
Reminder: Paths have to be simple...

Shortest Paths and Negative Cycles

Given $G = (V, E)$ with edge lengths and s, t. Suppose

1. G has a negative length cycle C, and
2. s can reach C and C can reach t.

Question: What is the shortest **distance** from s to t?

Possible answers: Define shortest distance to be:

1. undefined, that is $-\infty$, OR
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Really bad new about negative edges, and shortest path...

Lemma 18.3.

If there is an efficient algorithm to find a shortest simple $s \rightarrow t$ path in a graph with negative edge lengths, then there is an efficient algorithm to find the <u>longest</u> simple $s \rightarrow t$ path in a graph with positive edge lengths.

Finding the $s \rightarrow t$ longest path is difficult. **NP-Hard**!

THE END

...

(for now)