18.2
Bellman Ford Algorithm

18.2.1
Shortest path with negative lengths: The challenge

Shortest Paths with Negative Lengths

Lemma 18.1.

Let G be a directed graph with arbitrary edge lengths. If $s = v_0 \rightarrow v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_k$ is a shortest path from s to v_k then for $1 \leq i < k$:

1. *$s = v_0 \rightarrow v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_i$ is a shortest path from s to v_i*
2. *False: $dist(s, v_i) \leq dist(s, v_k)$ for $1 \leq i < k$. Holds true only for non-negative edge lengths.*

Cannot explore nodes in increasing order of distance! We need other strategies.

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THE END

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(for now)