Polynomial Time Reductions

Lecture 21

Tuesday, November 17, 2020

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21.1
A quick review: Polynomials

What is a polynomial

A **polynomial** is a function of the form:

$$f(x) = \sum_{i=0}^{t} a_i x^i.$$

For our purposes, we can assume that $a_i \geq 0$, for all i.
A term $a_k x^t$ is a **monomial**.
The **degree** of $f(x)$ is t.
We have $f(n) = O(n^t)$.

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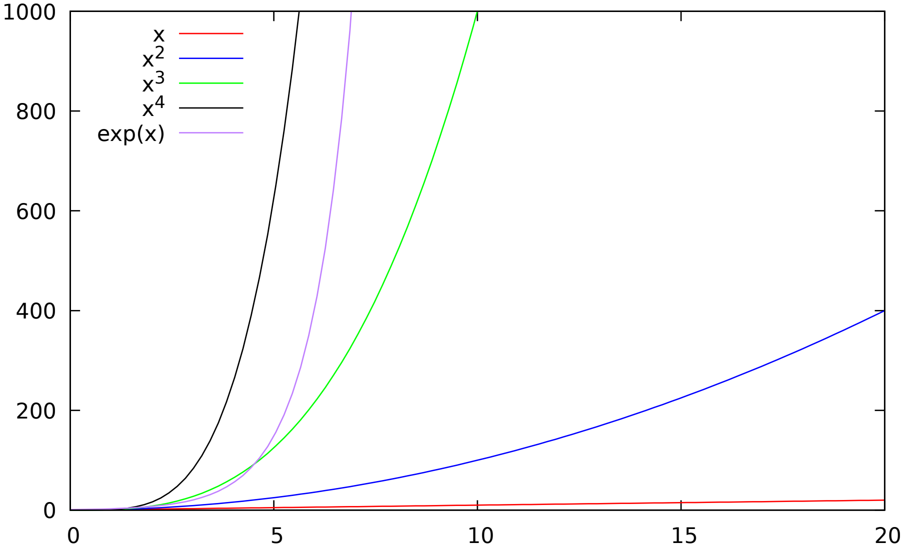
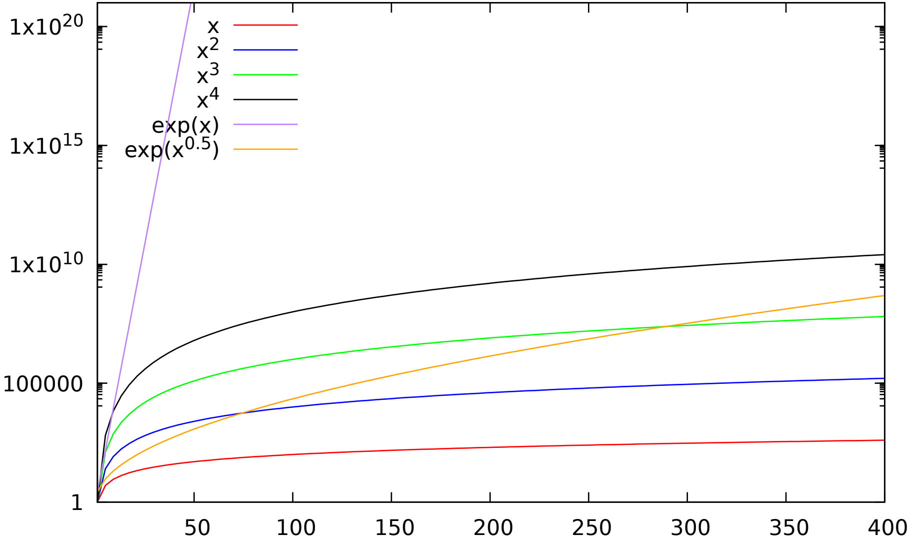
The degree of he polynomial matter...



Polynomial time good, exponential time bad



Combining polynomials

Lemma 21.1.

If $f(x) = \sum_{i=0}^{d} \alpha_i x^i$ is a polynomial of degree d, and $g(y) = \sum_{i=0}^{d'} \beta_i y^i$ is a polynomial of degree d', then $g(f(x))$ is a polynomial of degree $d'd$.

Proof.

Observe that $(f(x))^2 = \sum_{i=0}^{d} \sum_{j=0}^{d} \alpha_i \alpha_j x^{i+j}$ is a polynomial of degree $2d$, Arguing similarly, we have that $(f(x))^i$ is a polynomial of degree $i \cdot d$. Thus

$$g(f(x)) = \sum_{i=0}^{d'} \beta_i (f(x))^i$$

is a sum of polynomials of degree $0, d, 2d, \ldots, d \cdot d'$, which is a polynomial of degree $d \cdot d'$ by collecting monomials of the same degree into a single monomial. □

THE END

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(for now)