

Consider an NFA  $N = (Q, \Sigma, \delta, s, A)$ . A standard mental exercise is to try and negate it. Namely, consider the NFA  $\bar{N} = (Q, \Sigma, \delta, s, Q \setminus A)$ .

1.  $L(\bar{N}) = \overline{L(N)}$ :



$$L(N) = \{0,1\}^*, \text{ and } L(\bar{N}) = \emptyset.$$

2.  $L(N) \subsetneq L(\bar{N})$ :



$$L(N) = (0+1)^*1(0+1)^*, \text{ and } L(\bar{N}) = \Sigma^*.$$

3.  $L(N) \not\subseteq L(\bar{N})$ :



$$L(N) = \Sigma^* \text{ and } L(\bar{N}) = (0+1)^*1(0+1)^*.$$

In conclusion, there is no meaningful relation between  $L(N)$  and  $L(\bar{N})$ . As such, negating an NFA directly does not lead to an NFA for the complement language.