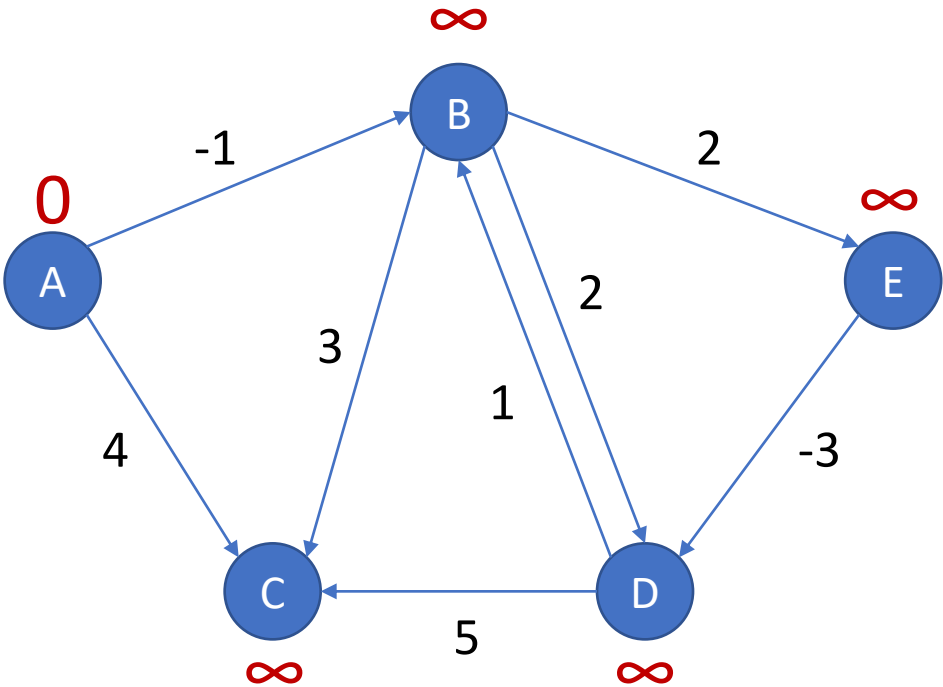


# Bellman-Ford Algorithm

## Example

Initialization:



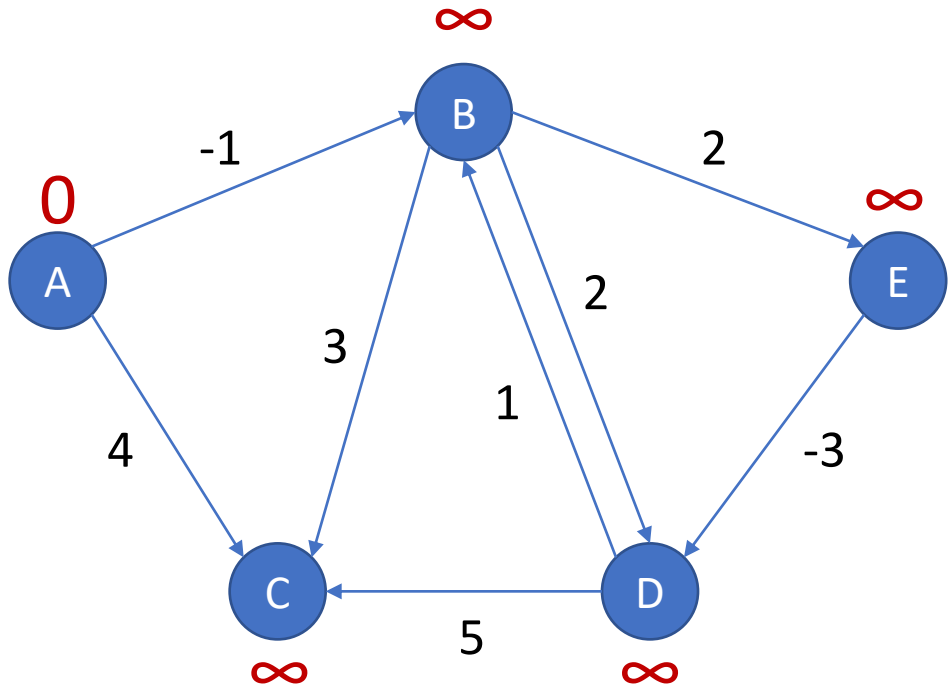
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=1:



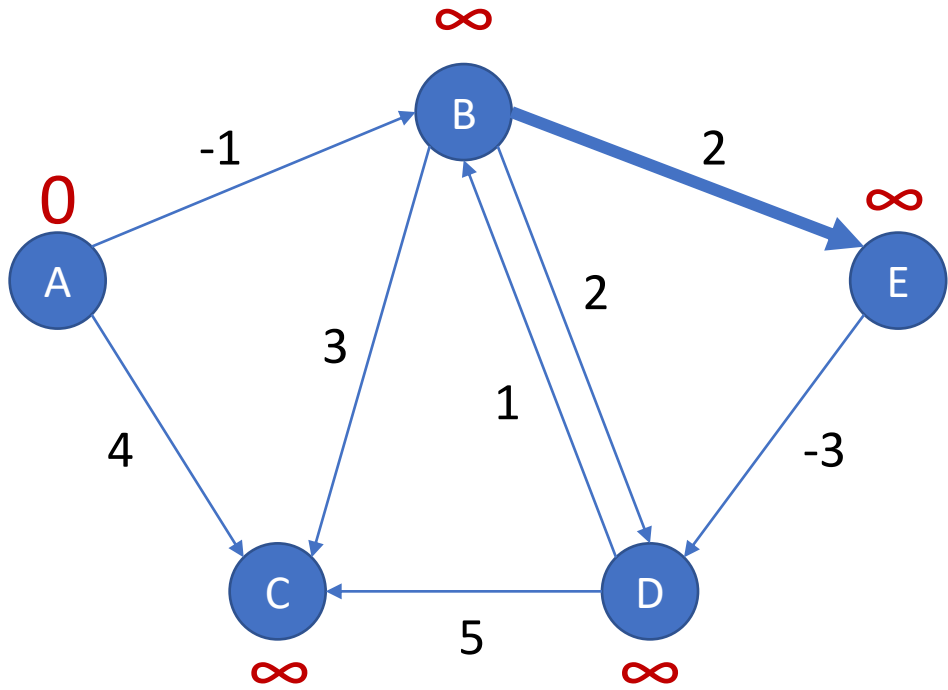
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=1:



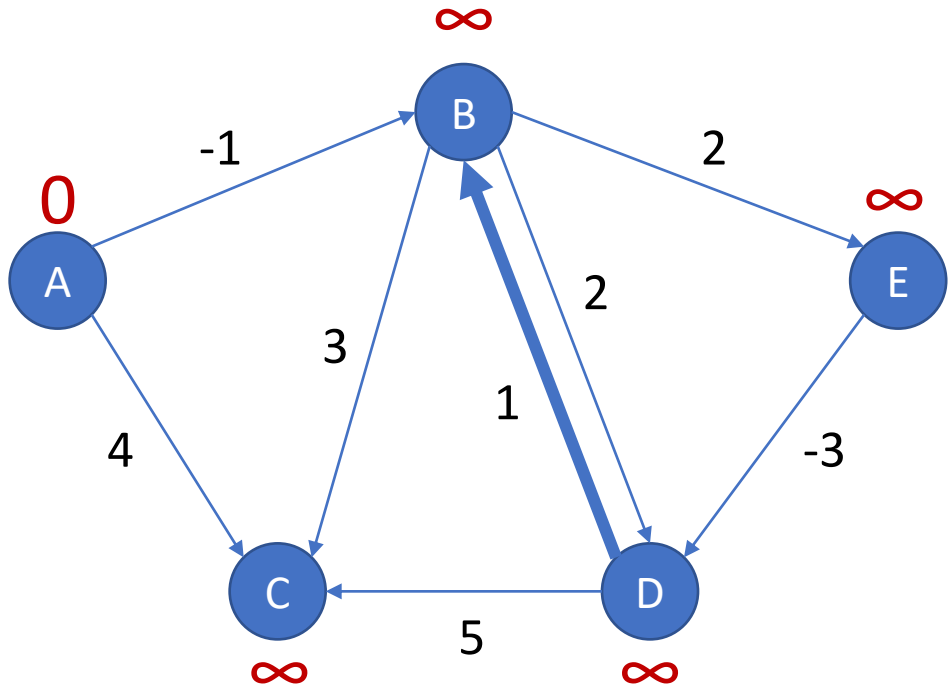
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=1:



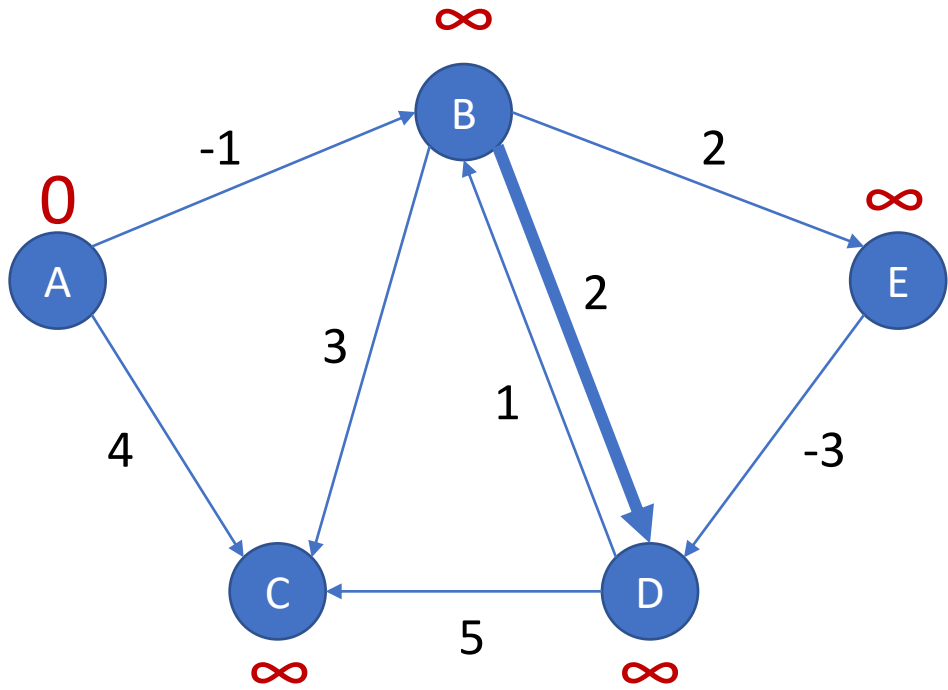
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=1:



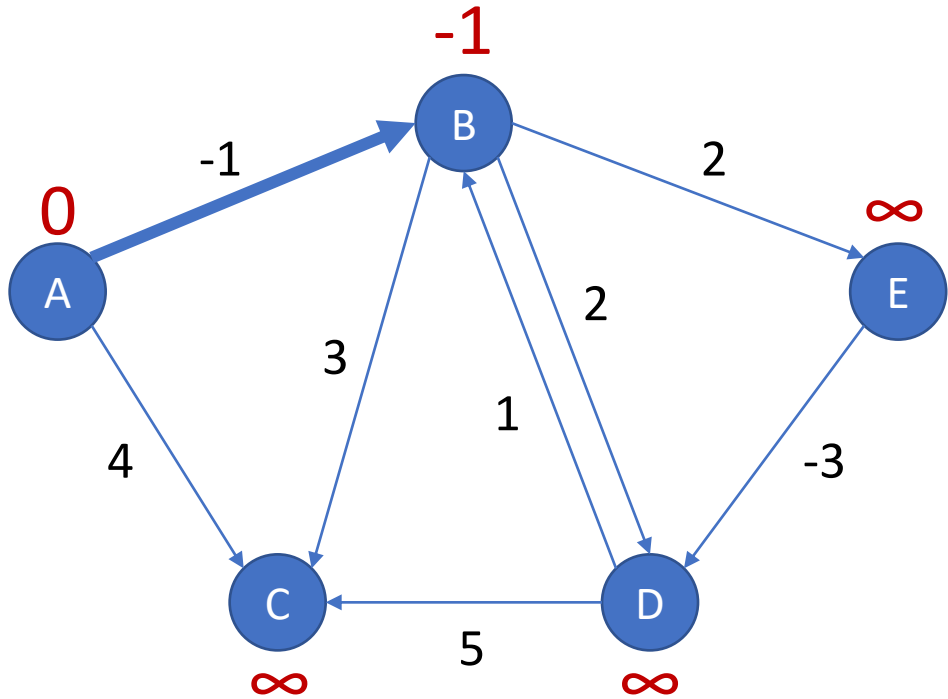
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=1:



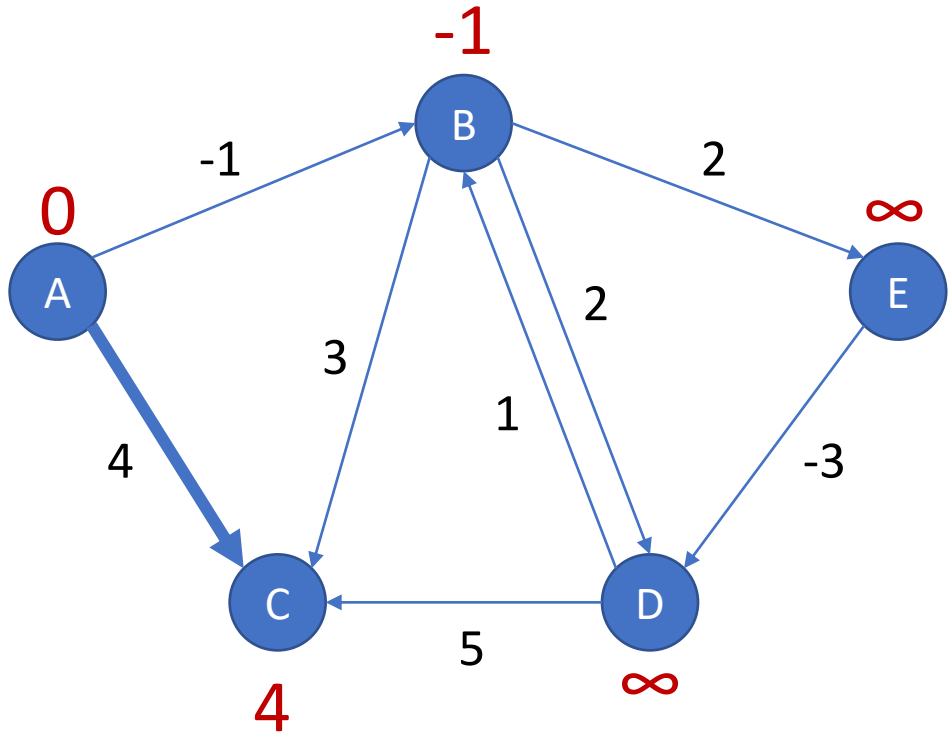
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=1:



	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$

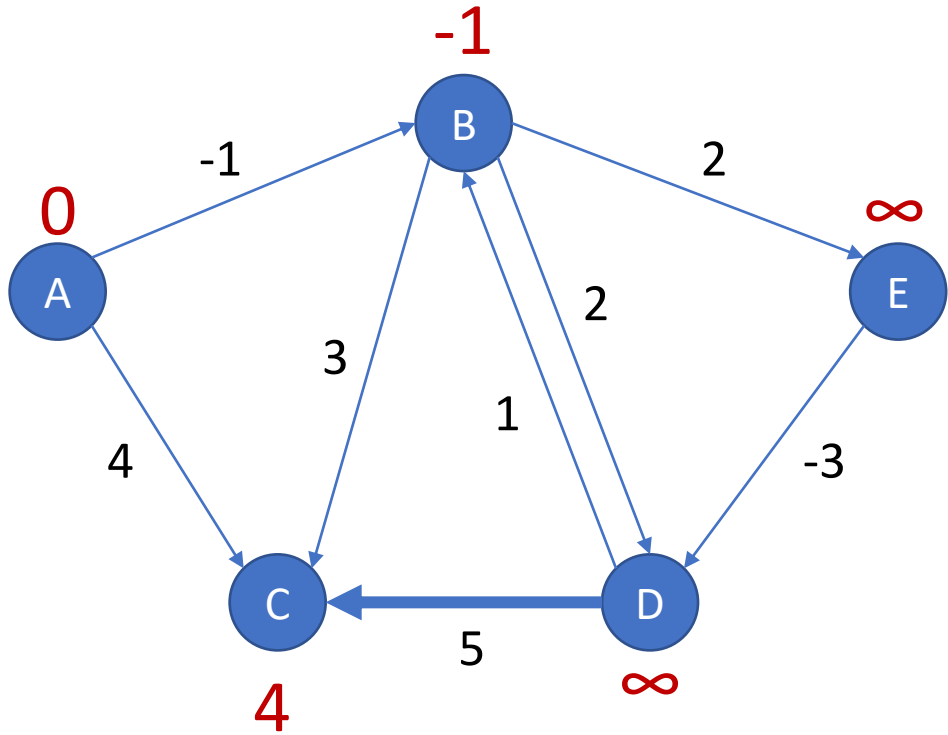
```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```



k=1:



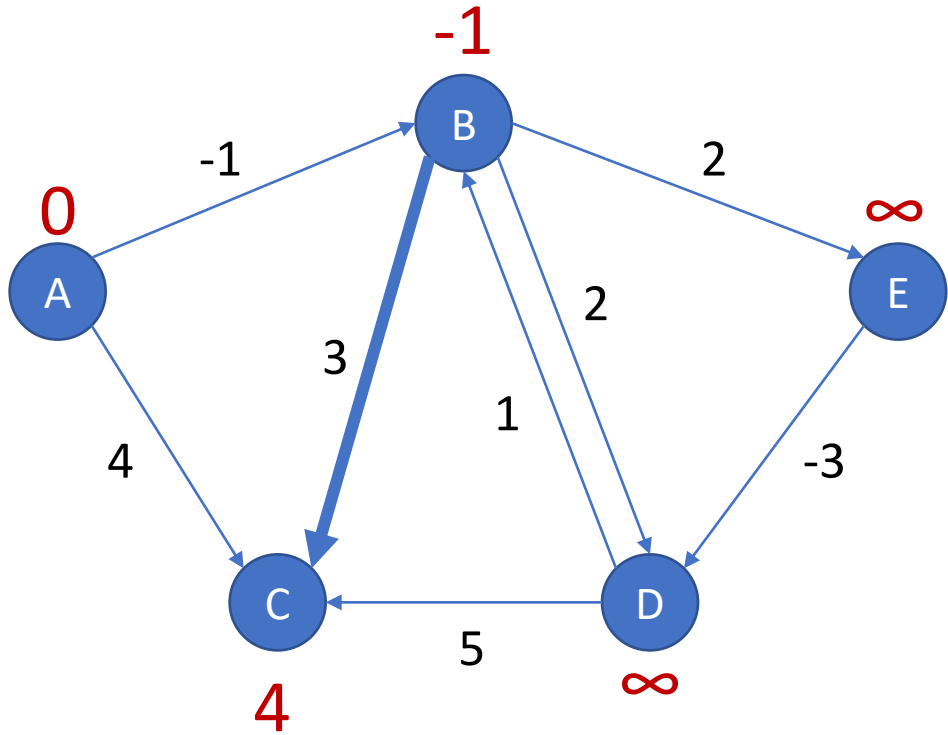
	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=1:



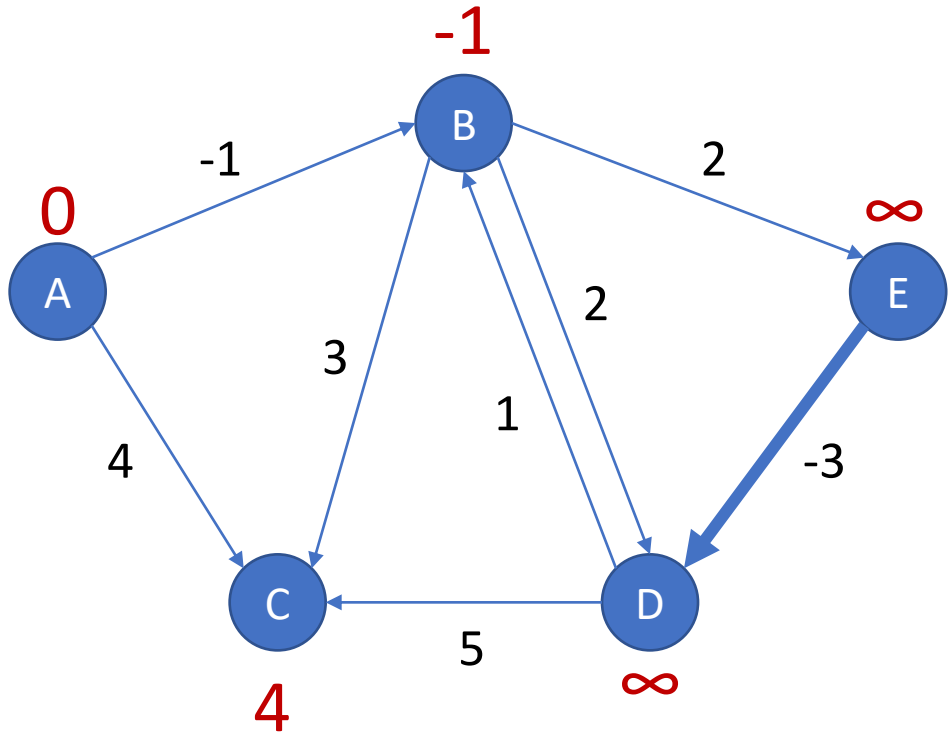
	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=1:



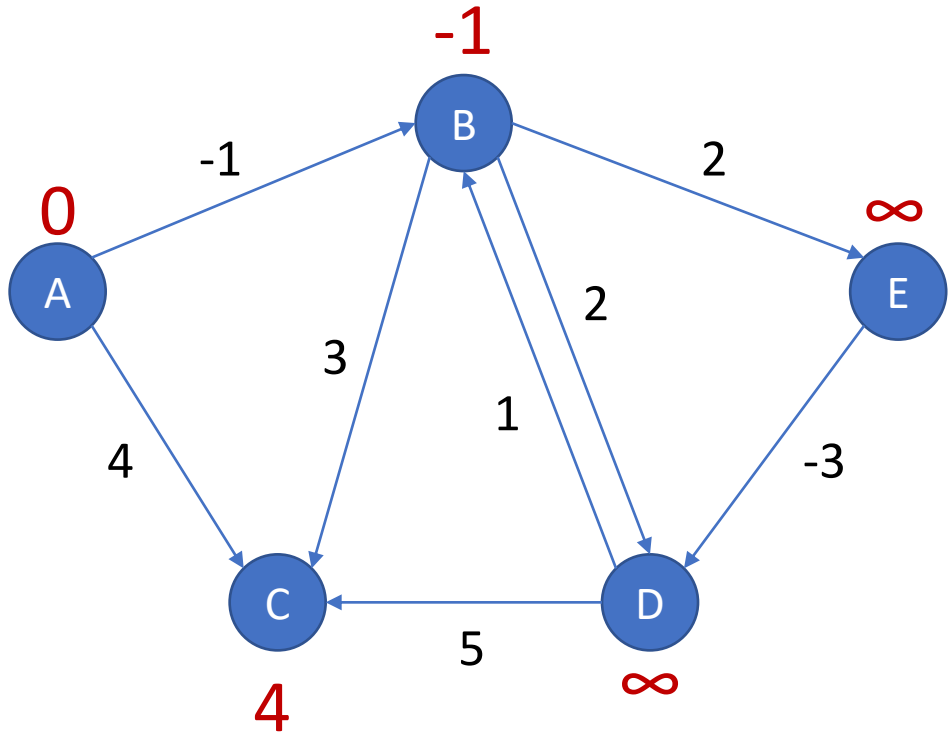
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=1:



A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$

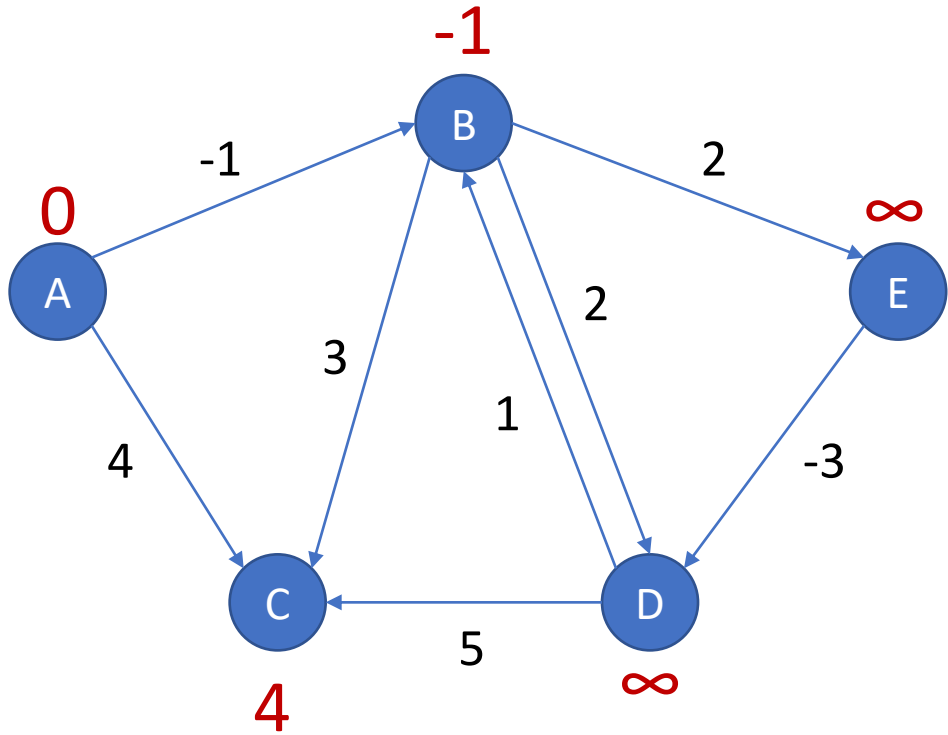
```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

Distances assuming all paths lengths are at most 1 edge long.

k=2:



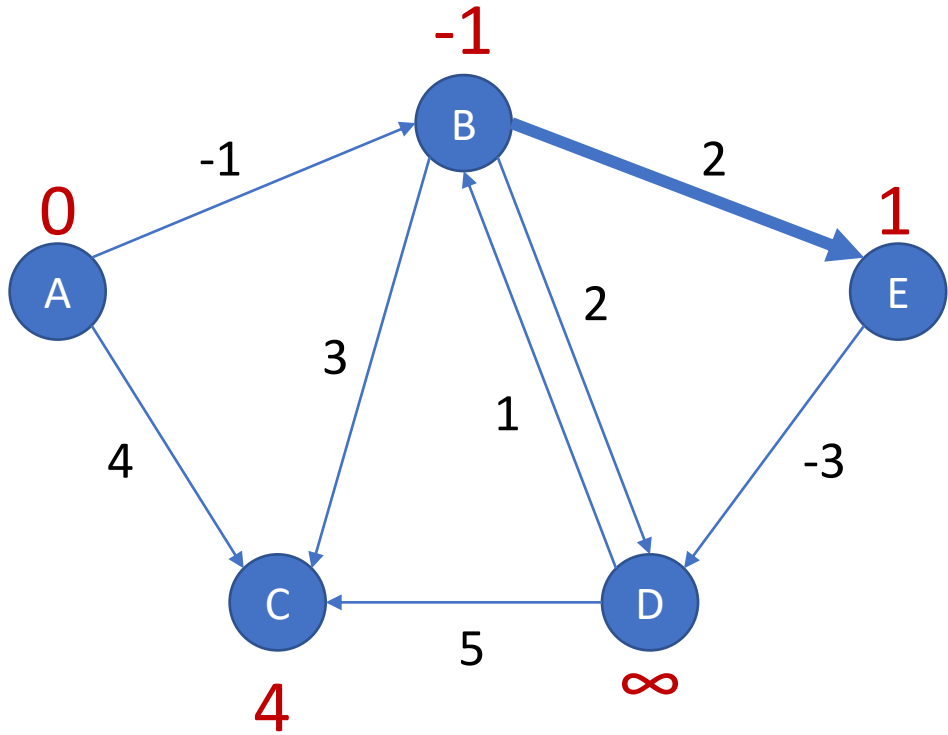
	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$
E	0	-1	4	$\infty$	$\infty$

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=2:



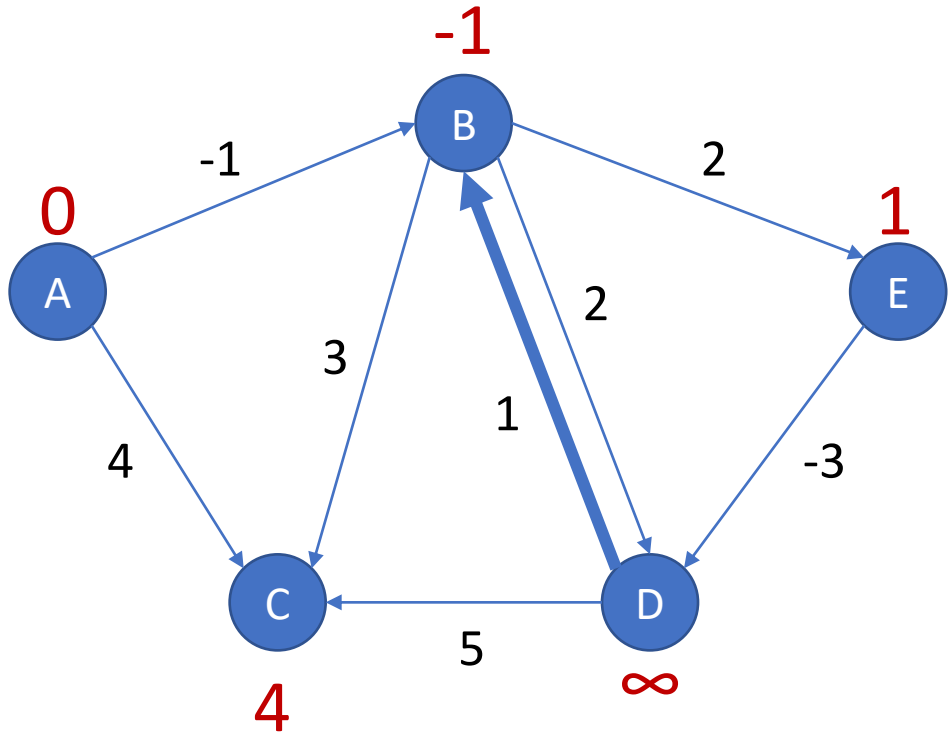
A	B	C	D	E
0	∞	∞	∞	∞
0	∞	∞	∞	∞
0	-1	∞	∞	∞
0	-1	4	∞	∞
0	-1	4	∞	∞
0	-1	4	∞	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=2:



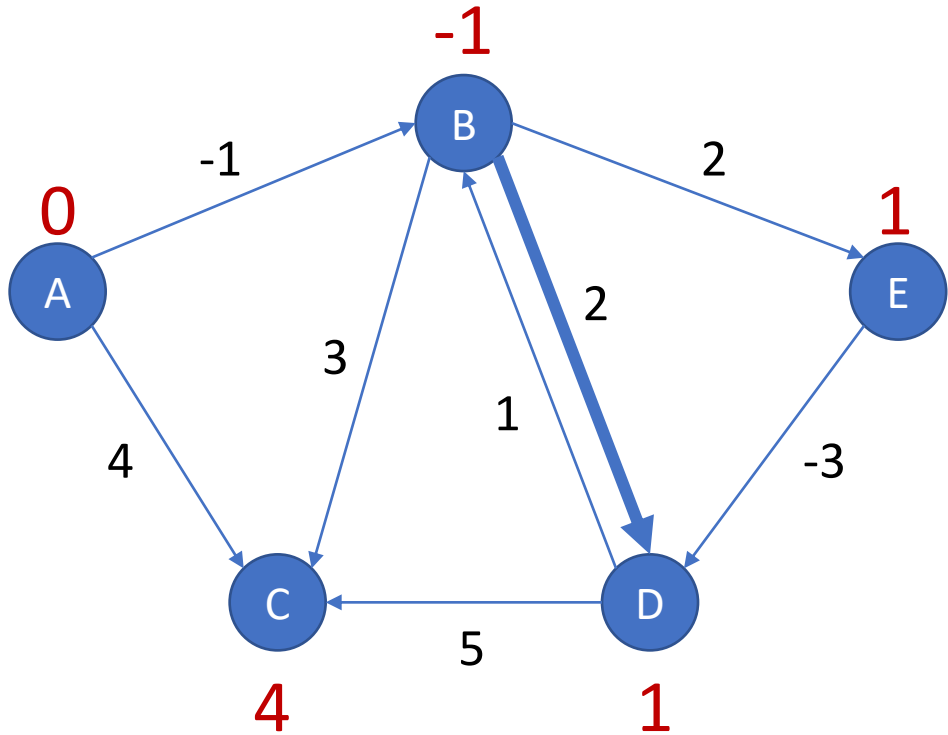
A	B	C	D	E
0	∞	∞	∞	∞
0	∞	∞	∞	∞
0	-1	∞	∞	∞
0	-1	4	∞	∞
0	-1	4	∞	∞
0	-1	4	∞	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=2:



A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	1
0	-1	4	1	1

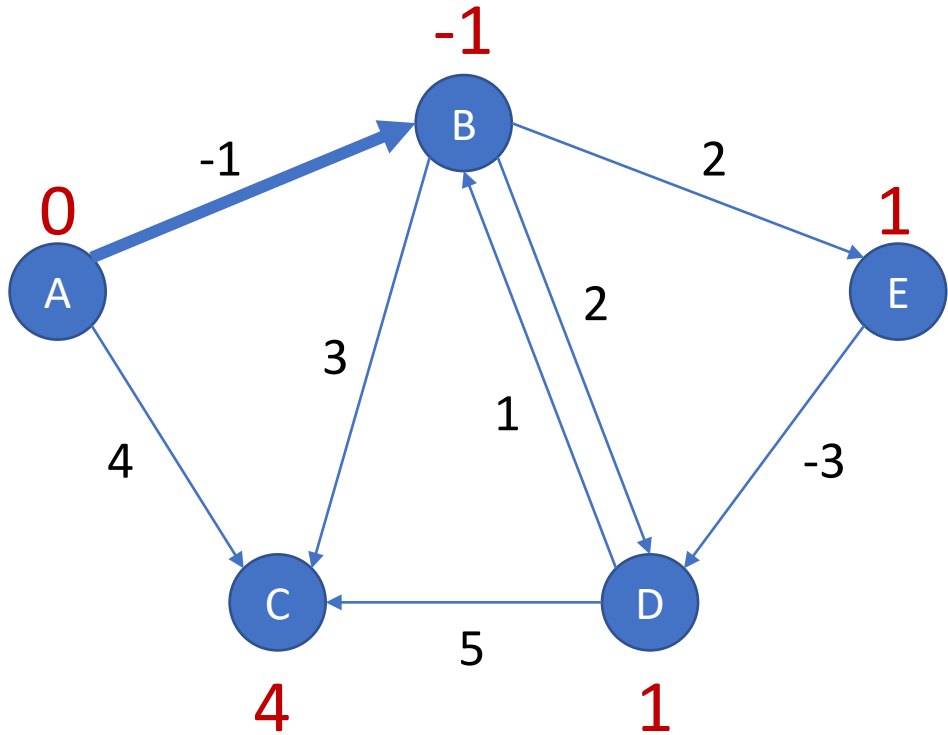
```
for each u in V
  d(u,0) = inf
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```



k=2:



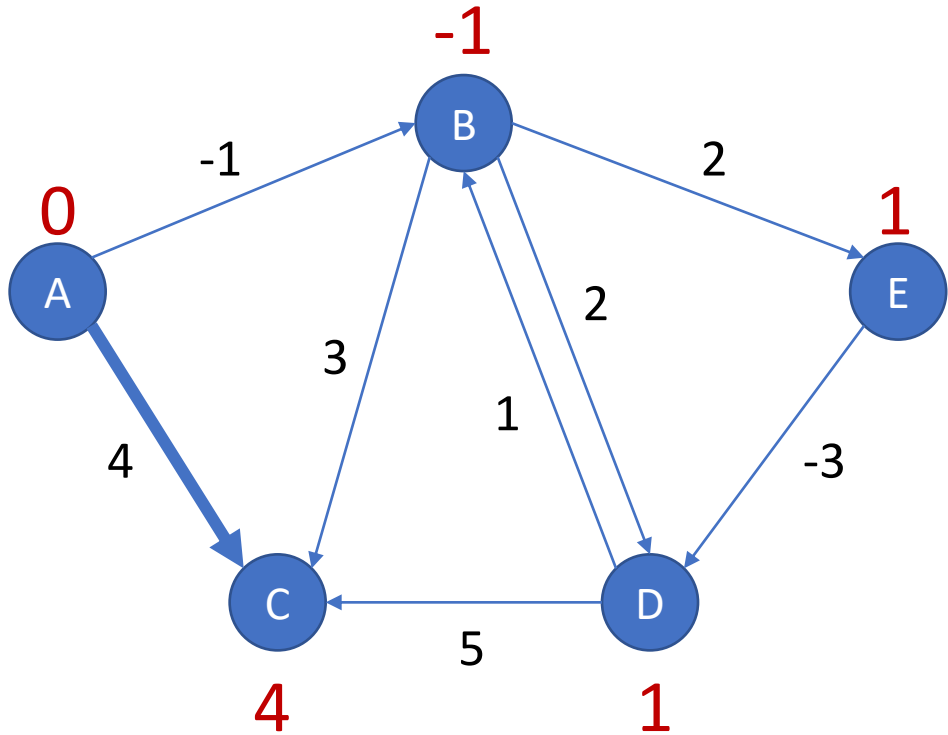
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	1
0	-1	4	1	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=2:



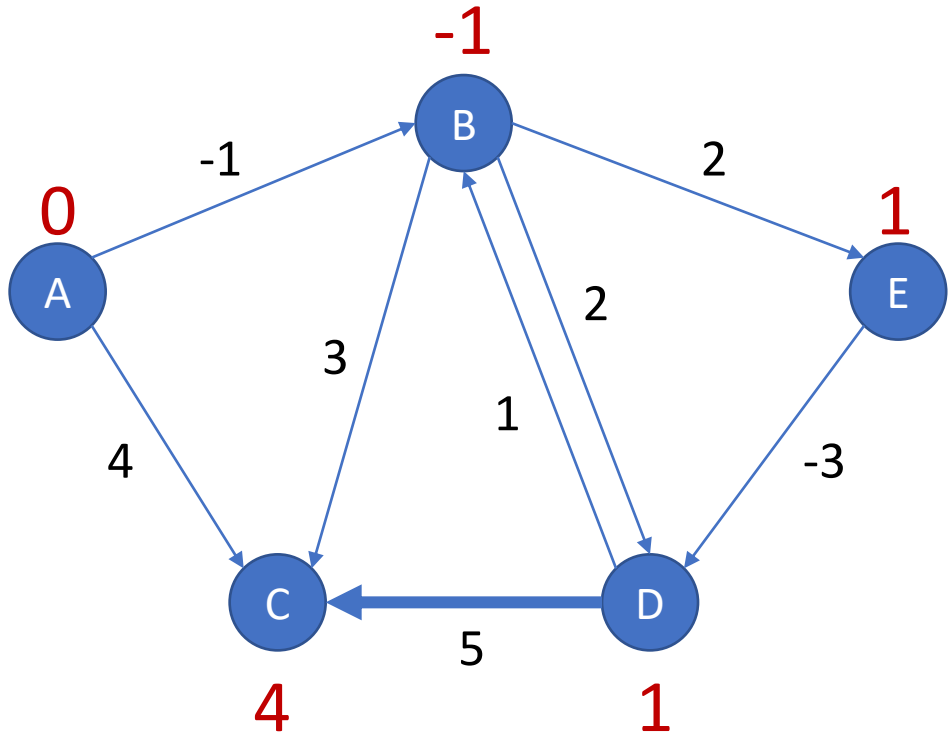
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	1
0	-1	4	1	1

```
for each u in V
  d(u,0) = inf
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=2:



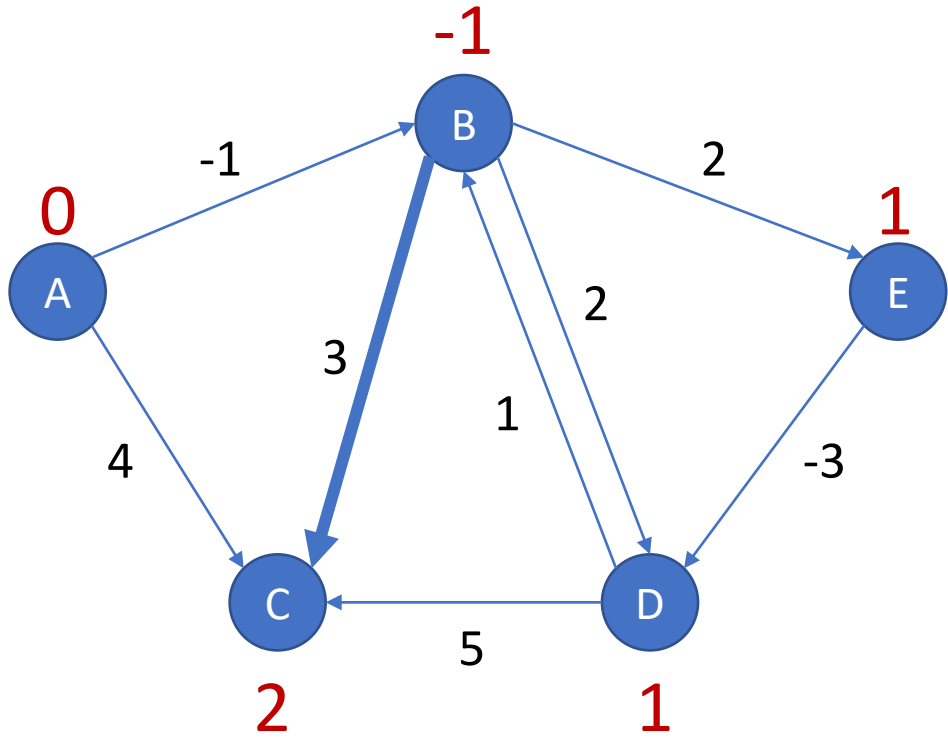
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	1
0	-1	4	1	1

```
for each u in V
  d(u,0) = inf
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=2:



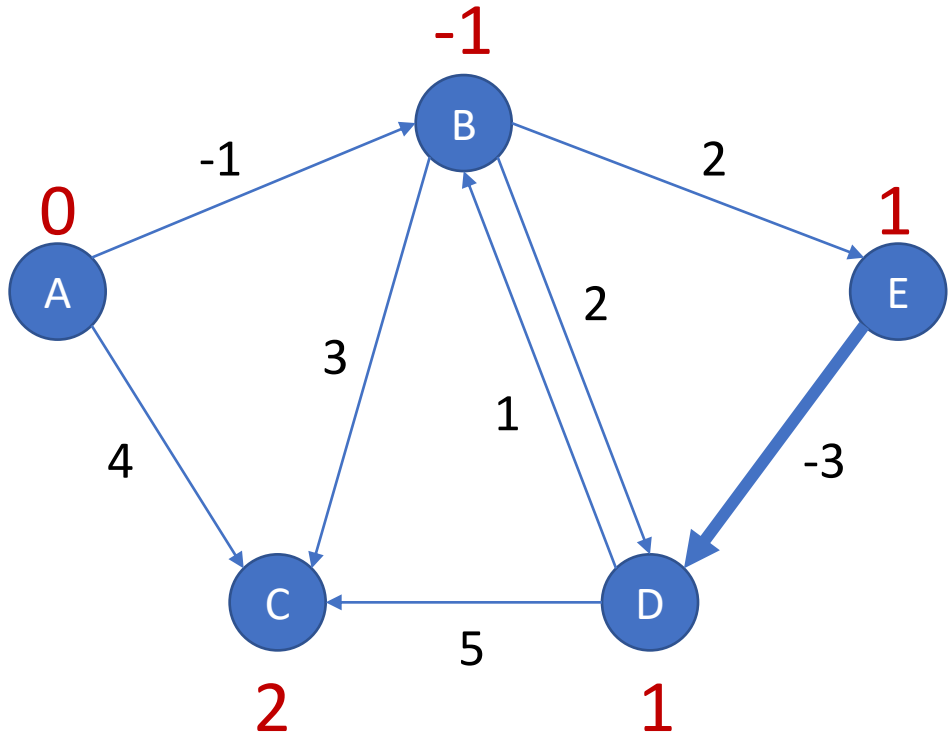
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	1
0	-1	4	1	1
0	-1	2	1	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=2:



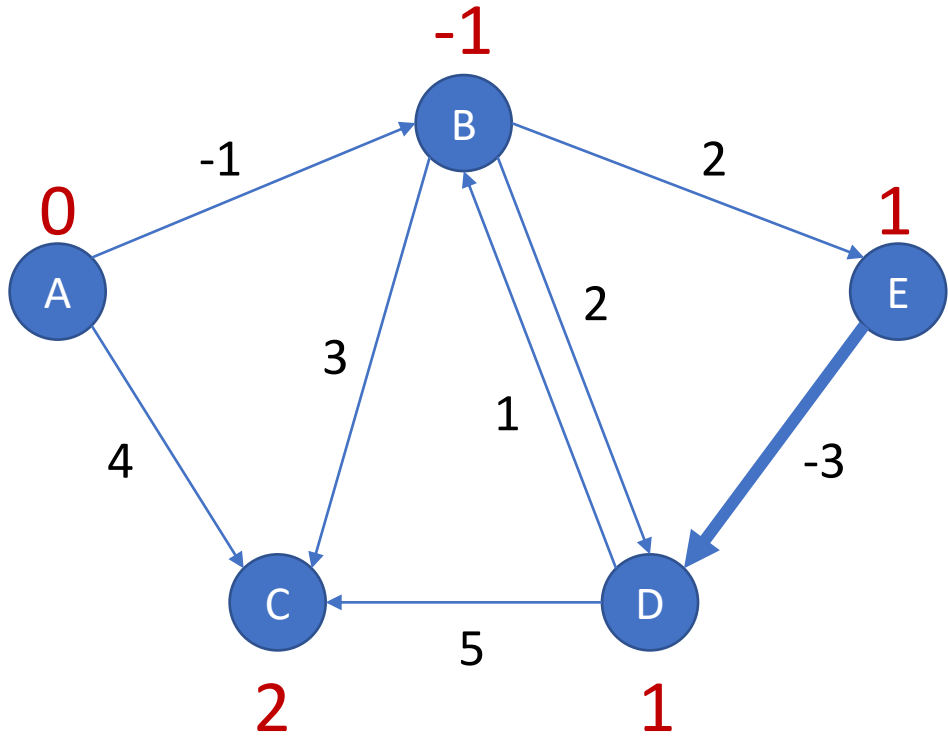
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	1
0	-1	4	1	1
0	-1	2	1	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=2:



A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	1	1
0	-1	2	1	1

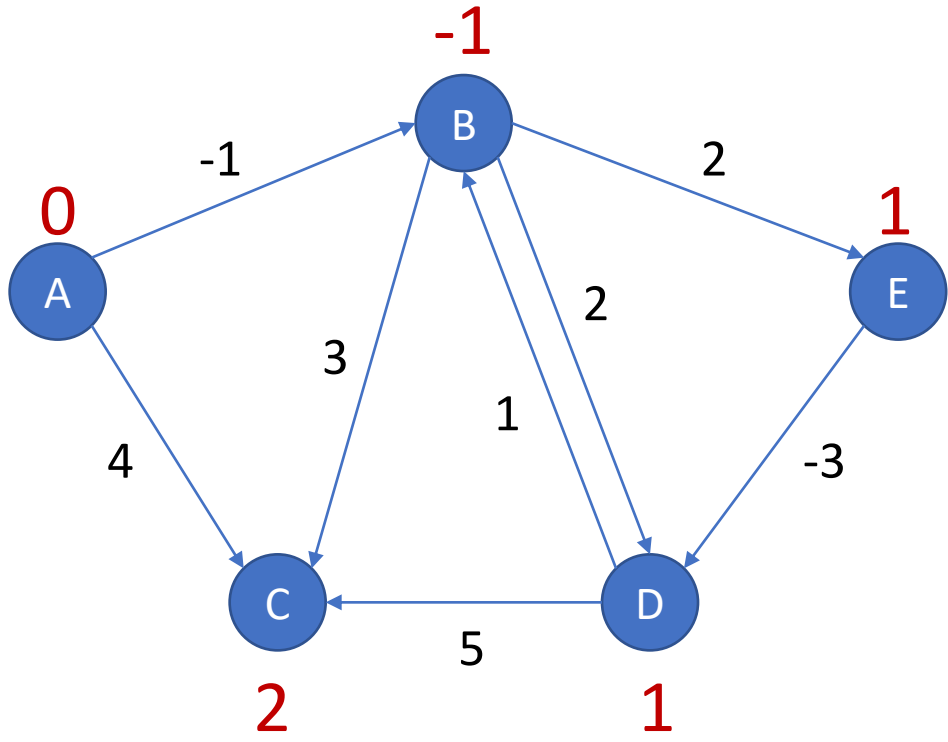
```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

Distances assuming all paths lengths are at most **2 edges long**.

k=3:



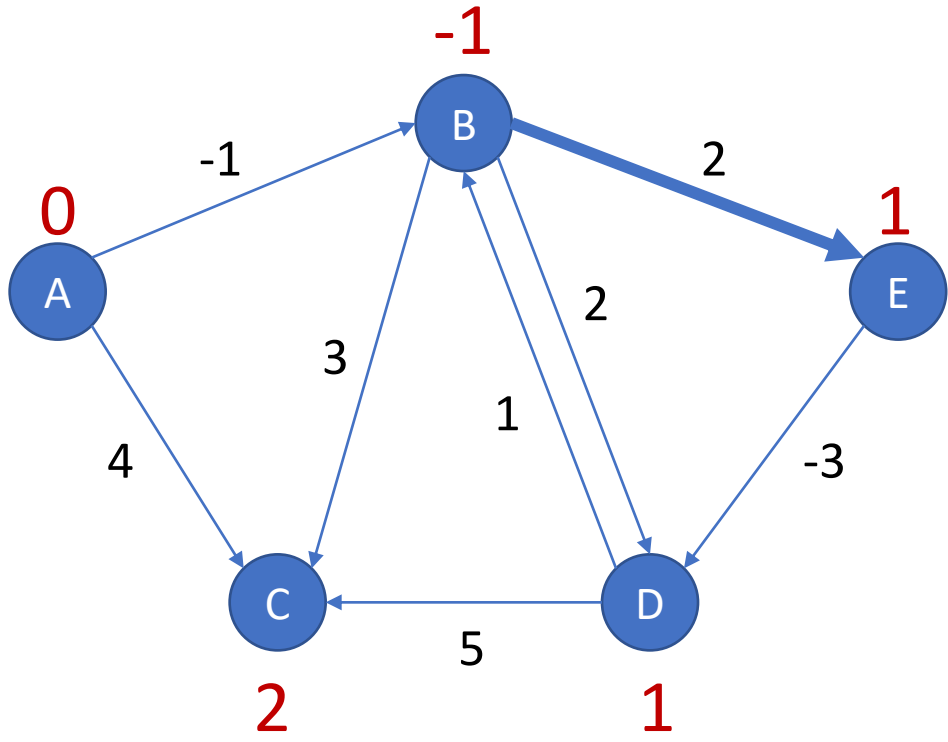
	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$
E	0	-1	4	$\infty$	$\infty$
	0	-1	4	$\infty$	1
	0	-1	4	1	1
	0	-1	2	1	1
	0	-1	2	1	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=3:



	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$
E	0	-1	4	$\infty$	$\infty$
	0	-1	4	$\infty$	1
	0	-1	4	1	1
	0	-1	2	1	1
	0	-1	2	1	1

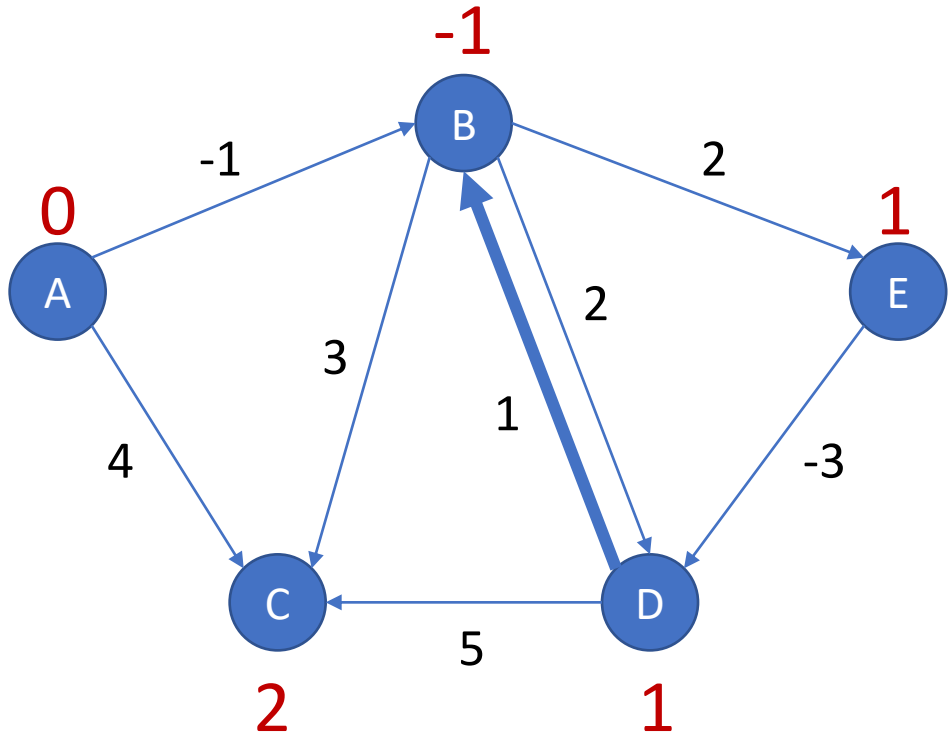
```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```



k=3:



	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$
E	0	-1	4	$\infty$	$\infty$
	0	-1	4	$\infty$	1
	0	-1	4	1	1
	0	-1	2	1	1
	0	-1	2	1	1

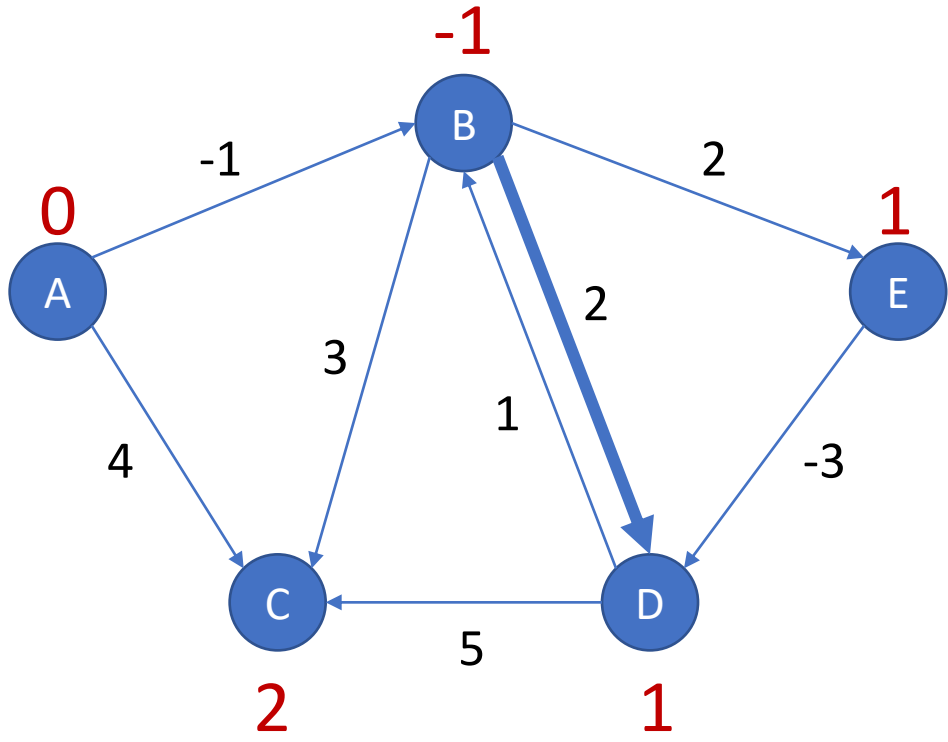
```

for each u in V
    d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
    for each v in V
        d(v,k) = d(v, k-1)
        for edge (u,v) in incoming(v)
            d(v,k) = min{d(v,k-1),
                        d(u,k-1)+l(u,v)}

for each v in V do
    dist(s,v) = d(v, n-1)
    
```

k=3:



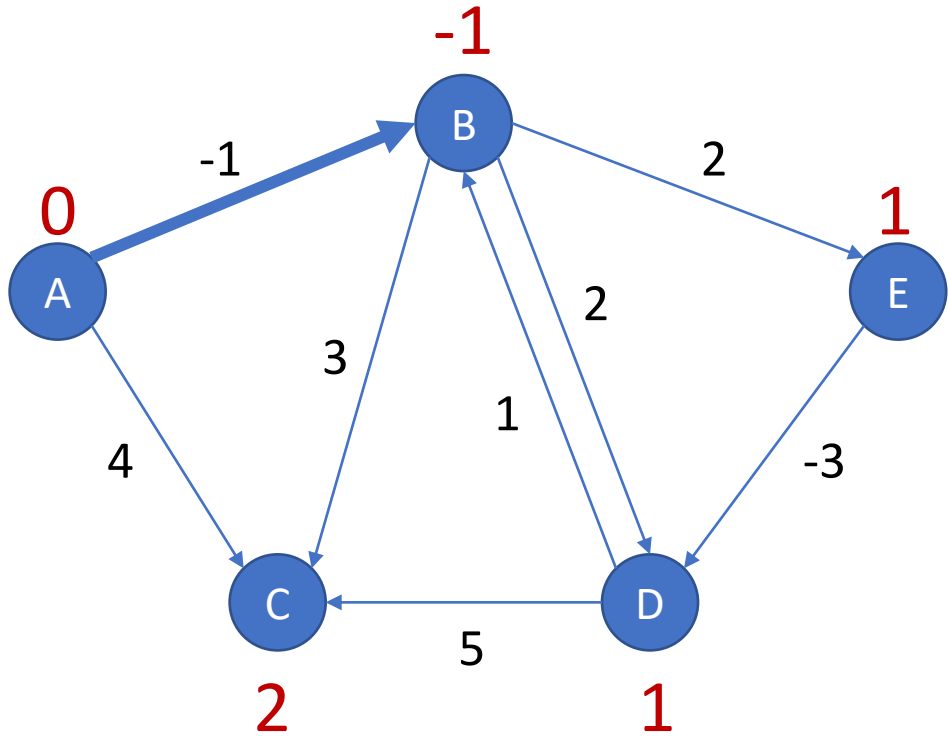
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	1
0	-1	4	1	1
0	-1	2	1	1
0	-1	2	1	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=3:



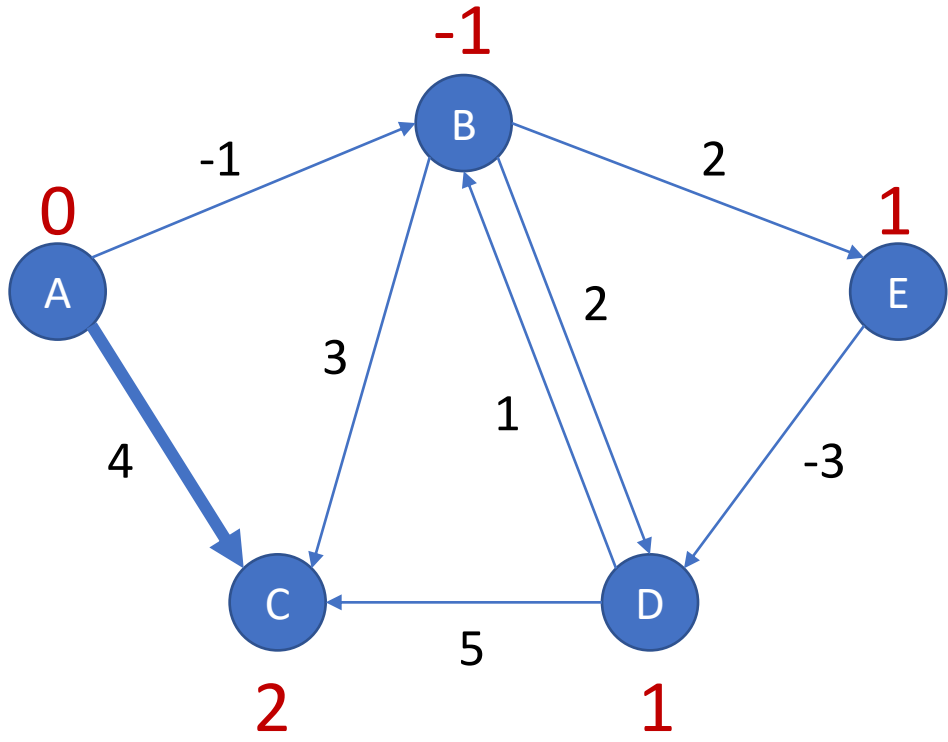
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	1
0	-1	4	1	1
0	-1	2	1	1
0	-1	2	1	1

```
for each u in V
  d(u,0) = infity
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=3:



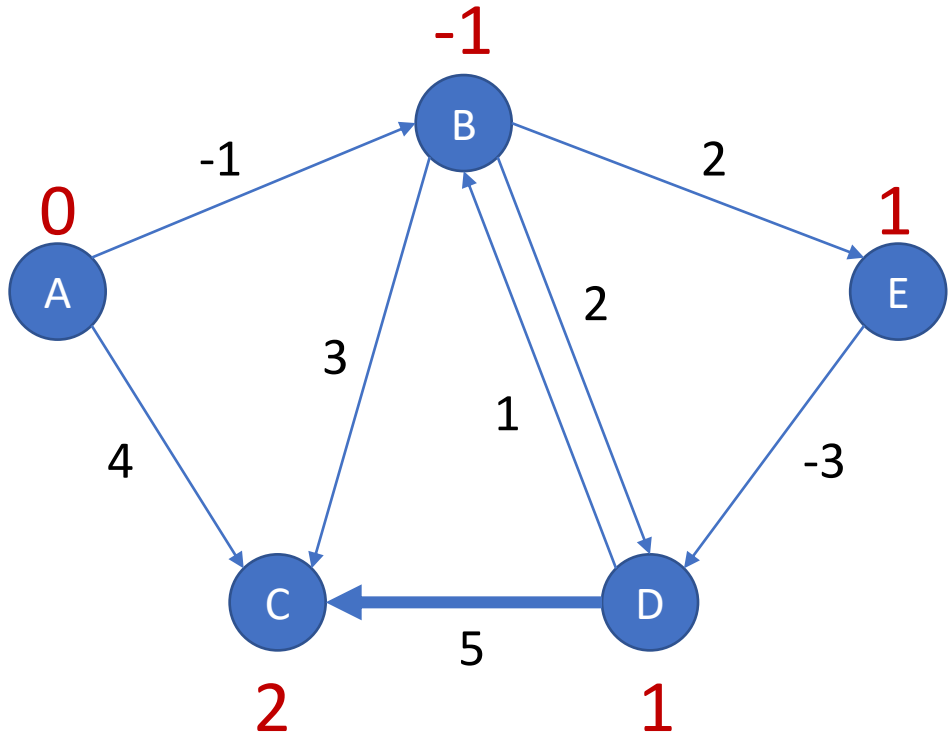
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	1
0	-1	4	1	1
0	-1	2	1	1
0	-1	2	1	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=3:



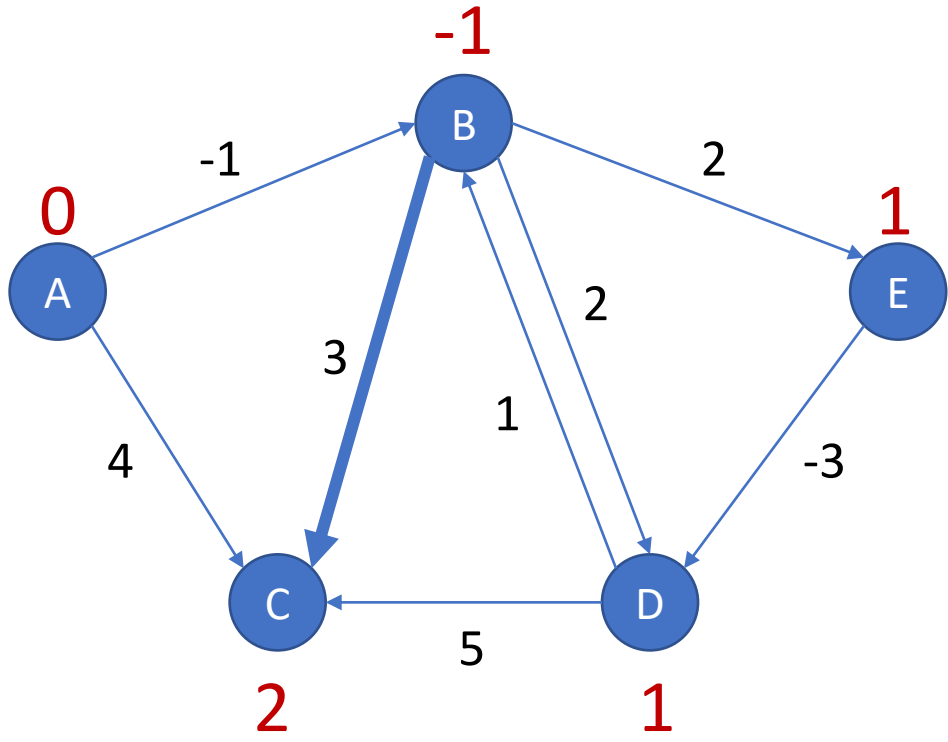
	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$
E	0	-1	4	$\infty$	$\infty$
	0	-1	4	$\infty$	1
	0	-1	4	1	1
	0	-1	2	1	1
	0	-1	2	1	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=3:



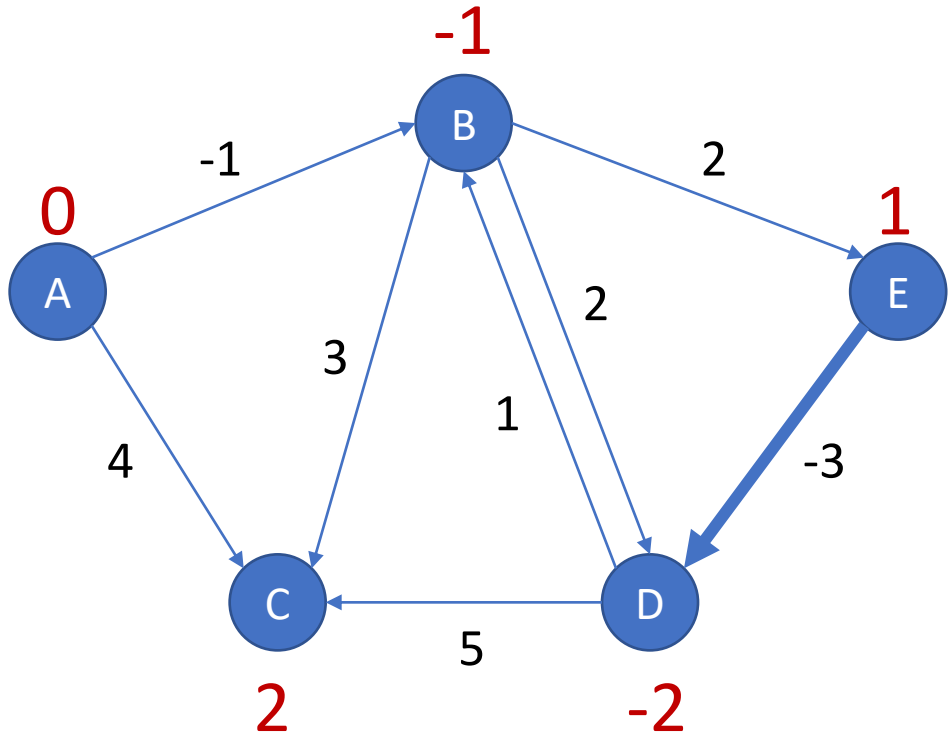
	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$
E	0	-1	4	$\infty$	$\infty$
	0	-1	4	$\infty$	1
	0	-1	4	1	1
	0	-1	2	1	1
	0	-1	2	1	1

```
for each u in V
  d(u,0) = inf
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=3:



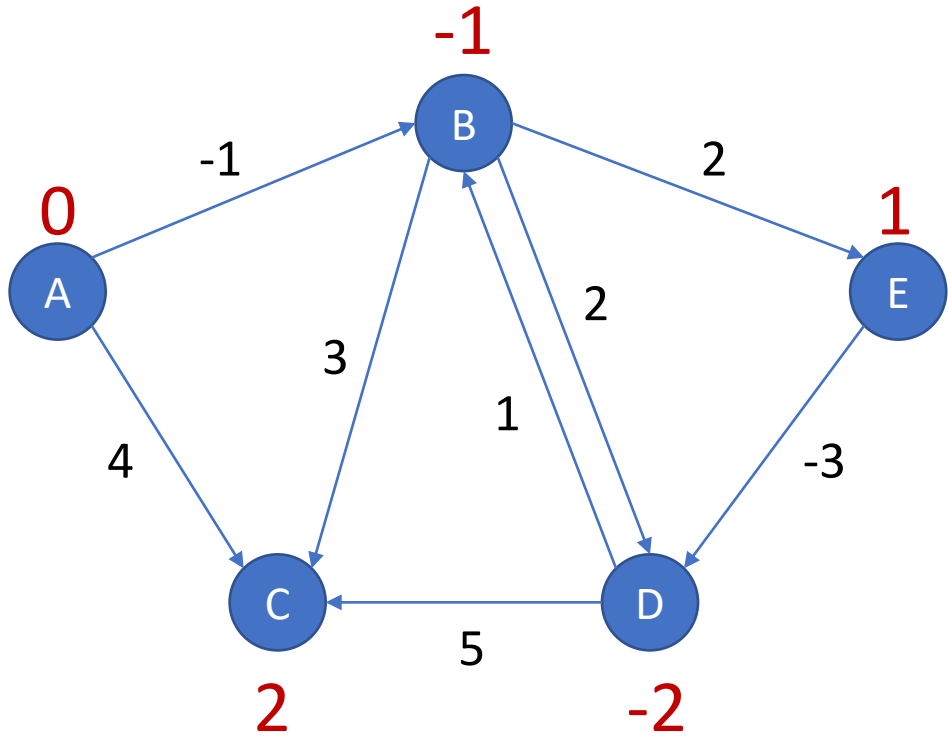
A	B	C	D	E
0	$\infty$	$\infty$	$\infty$	$\infty$
0	$\infty$	$\infty$	$\infty$	$\infty$
0	-1	$\infty$	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	$\infty$
0	-1	4	$\infty$	1
0	-1	4	1	1
0	-1	2	1	1
0	-1	2	1	1
0	-1	2	-2	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

k=3:



	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$
E	0	-1	4	$\infty$	$\infty$
	0	-1	4	$\infty$	1
	0	-1	4	1	1
	0	-1	2	1	1
	0	-1	2	1	1
	0	-1	2	-2	1

```
for each u in V
  d(u,0) = infty
d(s,0) = 0

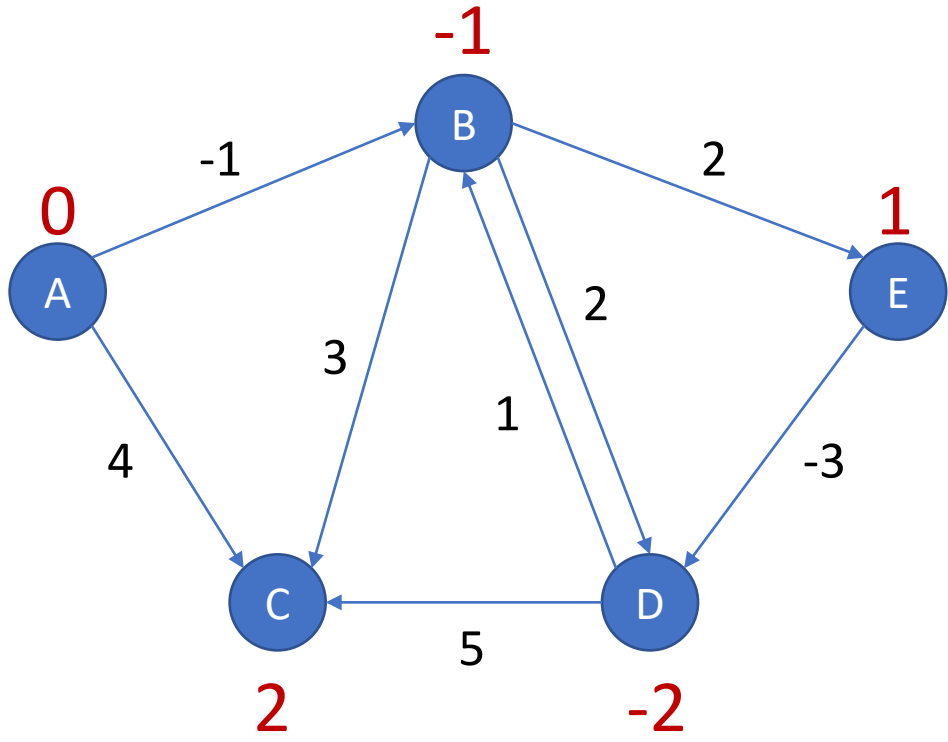
for k = 1 to n-1
  for each v in V
    d(v,k) = d(v, k-1)
    for edge (u,v) in incoming(v)
      d(v,k) = min{d(v,k-1),
                  d(u,k-1)+l(u,v)}

for each v in V do
  dist(s,v) = d(v, n-1)
```

Distances assuming all paths lengths are at most **3 edges long**.



k=4:



	A	B	C	D	E
A	0	$\infty$	$\infty$	$\infty$	$\infty$
B	0	$\infty$	$\infty$	$\infty$	$\infty$
C	0	-1	$\infty$	$\infty$	$\infty$
D	0	-1	4	$\infty$	$\infty$
E	0	-1	4	$\infty$	$\infty$
A	0	-1	4	$\infty$	1
B	0	-1	4	1	1
C	0	-1	2	1	1
D	0	-1	2	-2	1
E	0	-1	2	-2	1

```

for each u in V
    d(u,0) = infty
d(s,0) = 0

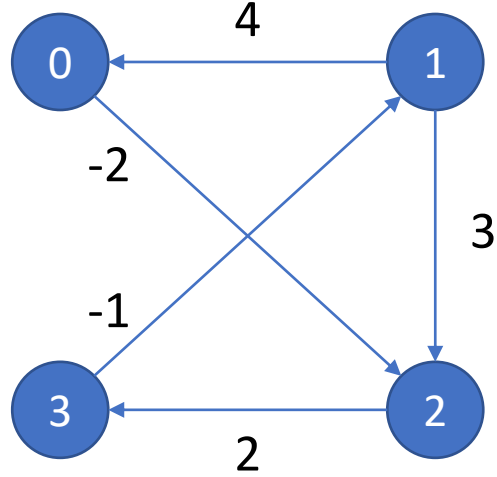
for k = 1 to n-1
    for each v in V
        d(v,k) = d(v, k-1)
        for edge (u,v) in incoming(v)
            d(v,k) = min{d(v,k-1),
                        d(u,k-1)+l(u,v)}

for each v in V do
    dist(s,v) = d(v, n-1)
    
```

Nothing changes on forth iteration.

# Floyd-Warshall Algorithm Example

Initialization:



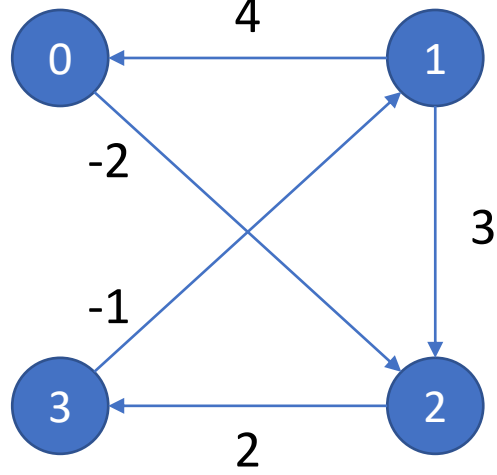
$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 3 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

```
for i = 1 to n
  for j = 1 to n
    d(i,j,0) = l(i,j)

for k = 0 to n
  for i = 0 to n
    for j = 0 to n
      d(i,j,k) = min{d(i,j,k-1),
                    d(i,k,k-1)+d(k,j,k-1)}

for i = 0 to n
  if dist(i,i,n) < 0
    print("Negative cycle detected!")
```

k=0:



$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 3 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

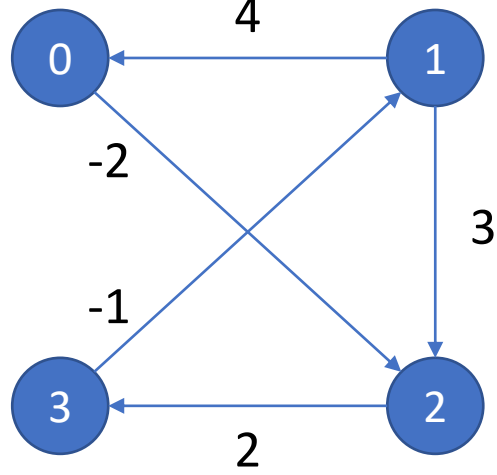
$$A^0 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 2 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

```
for i = 1 to n
  for j = 1 to n
    d(i,j,0) = l(i,j)
```

```
for k = 0 to n
  for i = 0 to n
    for j = 0 to n
      d(v,k) = min{d(i,j,k-1),
                  d(i,k,k-1)+d(k,j,k-1)}
```

```
for i = 0 to n
  if dist(i,i,n) < 0
    print("Negative cycle detected!")
```

k=0:



```
for i = 1 to n
  for j = 1 to n
    d(i,j,0) = l(i,j)

for k = 0 to n
  for i = 0 to n
    for j = 0 to n
      d(i,j,k) = min{d(i,j,k-1),
                    d(i,k,k-1)+d(k,j,k-1)}

for i = 0 to n
  if dist(i,i,n) < 0
    print("Negative cycle detected!")
```

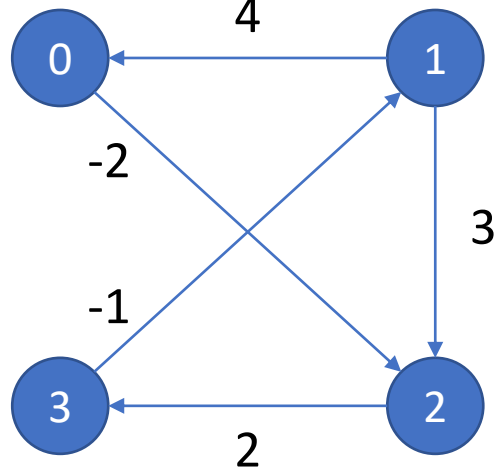
$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 3 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

$$A^0 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 2 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

$d(2,3,0) = \min \begin{cases} d(2,3,1) = 2 \\ d(2,0,1) + d(0,3,1) = \infty + \infty = \infty \end{cases}$

$d(1,2,0) = \min \begin{cases} d(1,2,1) = 3 \\ d(1,0,1) + d(0,2,1) = 4 + (-2) = 2 \end{cases}$

k=1:



$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 3 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

$$A^0 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 2 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

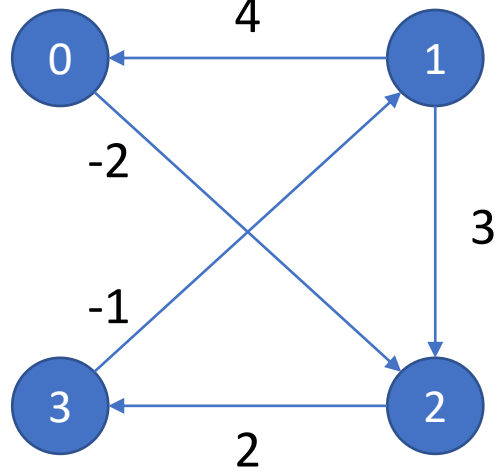
$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 2 & \infty \\ \infty & \infty & 0 & 2 \\ 3 & -1 & 1 & 0 \end{bmatrix}$$

```
for i = 1 to n
  for j = 1 to n
    d(i,j,0) = l(i,j)
```

```
for k = 0 to n
  for i = 0 to n
    for j = 0 to n
      d(i,j,k) = min{d(i,j,k-1),
                    d(i,k,k-1)+d(k,j,k-1)}
```

```
for i = 0 to n
  if dist(i,i,n) < 0
    print("Negative cycle detected!")
```

k=1:



$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 3 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

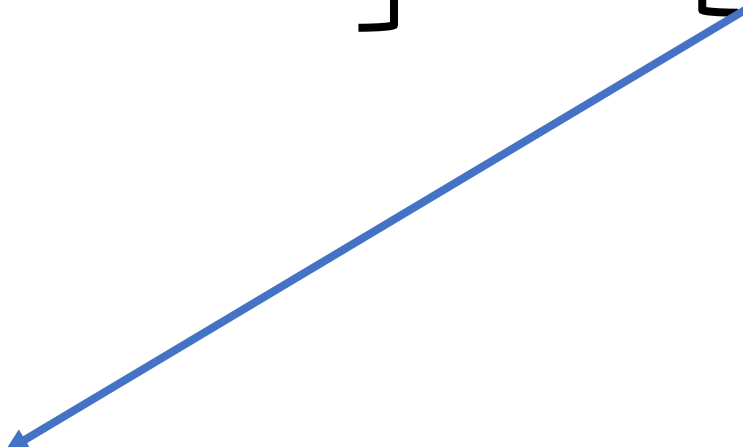
$$A^0 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 2 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 2 & \infty \\ \infty & \infty & 0 & 2 \\ 3 & -1 & 1 & 0 \end{bmatrix}$$

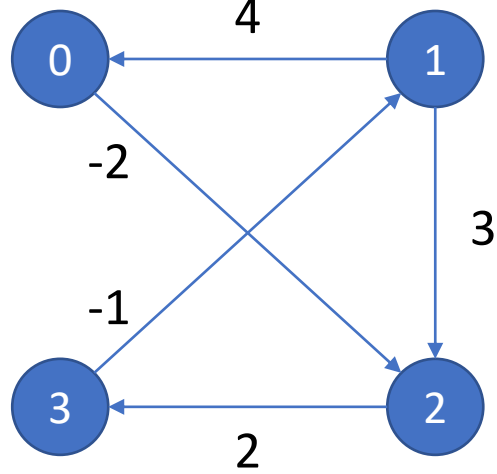
```
for i = 1 to n
  for j = 1 to n
    d(i,j,0) = l(i,j)

for k = 0 to n
  for i = 0 to n
    for j = 0 to n
      d(v,k) = min{d(i,j,k-1),
                  d(i,k,k-1)+d(k,j,k-1)}

for i = 0 to n
  if dist(i,i,n) < 0
    print("Negative cycle detected!")
```


$$d(3,0,1) = \min \begin{cases} d(3,0,0) = \infty \\ d(3,1,0) + d(1,0,0) = (-1) + (4) = 3 \end{cases}$$

k=2:



$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 3 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

$$A^0 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 2 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 2 & \infty \\ \infty & \infty & 0 & 2 \\ 3 & -1 & 1 & 0 \end{bmatrix}$$

```
for i = 1 to n
  for j = 1 to n
    d(i,j,0) = l(i,j)
```

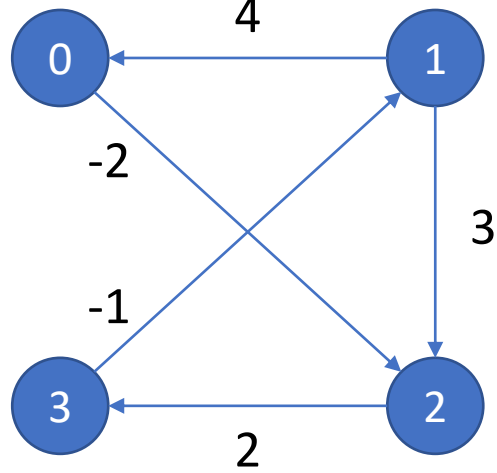
```
for k = 0 to n
  for i = 0 to n
    for j = 0 to n
      d(v,k) = min{d(i,j,k-1),
                  d(i,k,k-1)+d(k,j,k-1)}
```

```
for i = 0 to n
  if dist(i,i,n) < 0
    print("Negative cycle detected!")
```

$$A^2 = \begin{bmatrix} 0 & \infty & -2 & 0 \\ 4 & 0 & 2 & 4 \\ \infty & \infty & 0 & 2 \\ 3 & -1 & 1 & 0 \end{bmatrix}$$



k=3:



$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 3 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

$$A^0 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 2 & \infty \\ \infty & \infty & 0 & 2 \\ \infty & -1 & \infty & 0 \end{bmatrix}$$

$$A^1 = \begin{bmatrix} 0 & \infty & -2 & \infty \\ 4 & 0 & 2 & \infty \\ \infty & \infty & 0 & 2 \\ 3 & -1 & 1 & 0 \end{bmatrix}$$

```
for i = 1 to n
  for j = 1 to n
    d(i,j,0) = l(i,j)
```

```
for k = 0 to n
  for i = 0 to n
    for j = 0 to n
      d(v,k) = min{d(i,j,k-1),
                  d(i,k,k-1)+d(k,j,k-1)}
```

```
for i = 0 to n
  if dist(i,i,n) < 0
    print("Negative cycle detected!")
```

$$A^2 = \begin{bmatrix} 0 & \infty & -2 & 0 \\ 4 & 0 & 2 & 4 \\ \infty & \infty & 0 & 2 \\ 3 & -1 & 1 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & -1 & -2 & 0 \\ 4 & 0 & 2 & 4 \\ 5 & 1 & 0 & 2 \\ 3 & -1 & 1 & 0 \end{bmatrix}$$