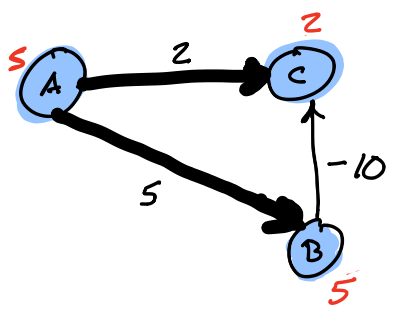
Lecture 20 Scribbles
Chat Moderators: Vasilis & Samir

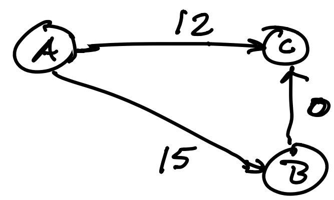
Topics:
- Shortest Paths w/ Negative Edges
- Bellman-Ford Algorithm
- Floyd-Warshall Algorithm

Dijkstra's Algorithm - gives shortest path from s to all other vertices

* can't tolerate negative edges

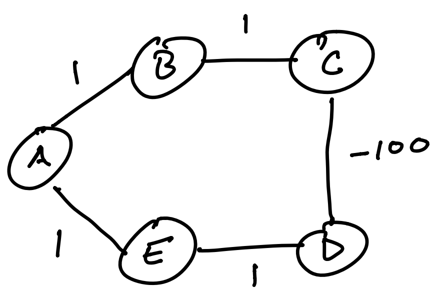


s = A

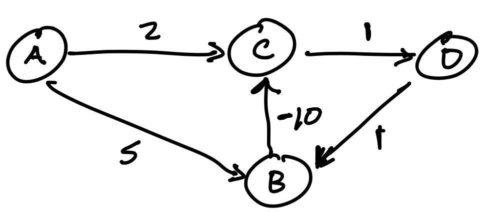


A → C ✗ 10

A → B → C ✗ 20



Negative Cycles



shortest walk from A → C

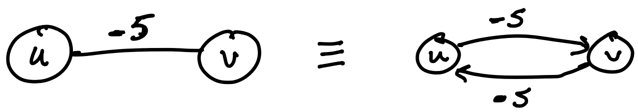
A - B - C - D - B - C - D - B - C

-5 -13 -21

A **path** is a sequence of distinct vertices such that $(v_{i-1}, v_i) \in E$

A **walk** is a sequence of vertices such that . . .

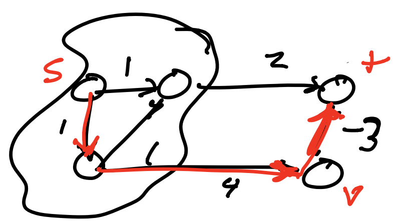
Converting directed from undirected graphs



Problem: Given graph $G = (V, E)$ which may have negative edges

1. Is there a negative cycle?
2. Given s find the shortest paths to all the other vertices.
3. Find the shortest path between all vertices.

2. Given s find the shortest paths to all the other vertices.



Observation: If we have a shortest path from $s \to v_k$ then $s \to v_{k-1}$ must also be a shortest path

Compute all the s.p.'s that use 1 edge
" 2 edge
3 edge

$d(v,k)$: shortest walk from s to v using at most k edges

$$d(v,k) = \min \begin{cases} \min_{u \in V} \left(d(u,k-1) + \ell(u,v) \right) & s \underset{k-1}{\rightsquigarrow} u \underset{1}{\to} v \\ d(v,k-1) & s \underset{k-1}{\rightsquigarrow} v \end{cases}$$

$s \underset{k}{\rightsquigarrow} v$

Base Case: $d(s,0) = 0$ & $d(v,0) = \infty$

All-Pairs Shortest Path

n Dijkstra $\to O(nm + n^2 \log n)$ assuming fancy heaps

n Bellman-Ford $\to O(n^2 m)$ $\quad m = n^2$

$\equiv O(n^4)$

Floyd-Warshall $\to O(n^3)$

Floyd-Warshall Algorithm

- if we # the vertices from 1 to n
- define $dist(i,j,k)$ = length of the shortest walk from v_i to v_j where the index of the intermediate node is at most k.

$$dist(i,j,k) = \min \begin{cases} dist(i,j,k-1) & i \underset{1 \text{ to } k-1}{\to} j \\ dist(i,k,k-1) + dist(k,j,k-1) & i \underset{v_1 \text{ to } k-1}{\to} k \underset{v_1 \text{ to } k-1}{\longrightarrow} j \end{cases}$$

Base Case: $dist(i,j,0) = \ell(i,j)$