Lecture 24 Scribble

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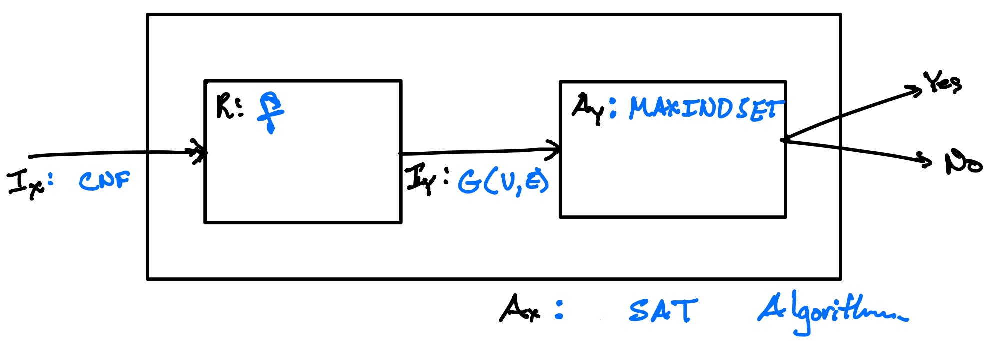
Topics:
- Sample Reduction
- Complexity Classes
- NP / NP-hard
- Certifiers / Verifiers

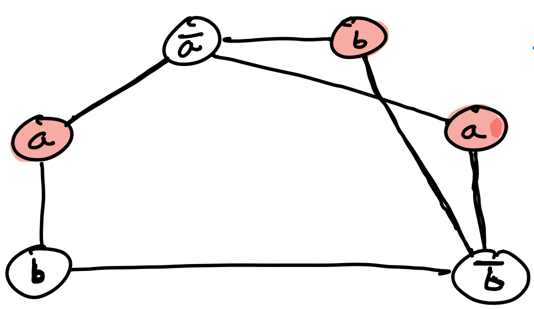
Example Reduction:

SAT: Given CNF formula φ
Q: Is φ satisfiable — NP-hard

MAX IND SET: Given $G(V,E)$
Q: Is there an independent set $\geq k$ — Assume NP-hard



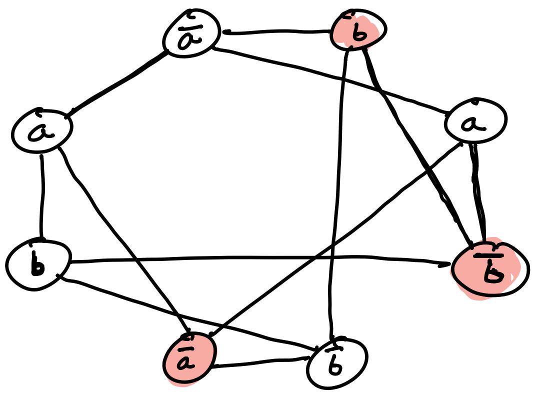
I_x Example: $(a \vee b) \wedge (\bar{a} \vee b) \wedge (a \vee \bar{b})$ — satisfied $a = b = 1$



$f(\varphi) = G(V,E)$
Create a graph G from SAT
- edge between vars in same clause
- edge between vars and their complements

If φ is sat then
I.S$[G] = k$ (# of clauses)

$(a \vee b) \wedge (\bar{a} \vee b) \wedge (a \vee \bar{b}) \wedge (\bar{a} \vee \bar{b})$

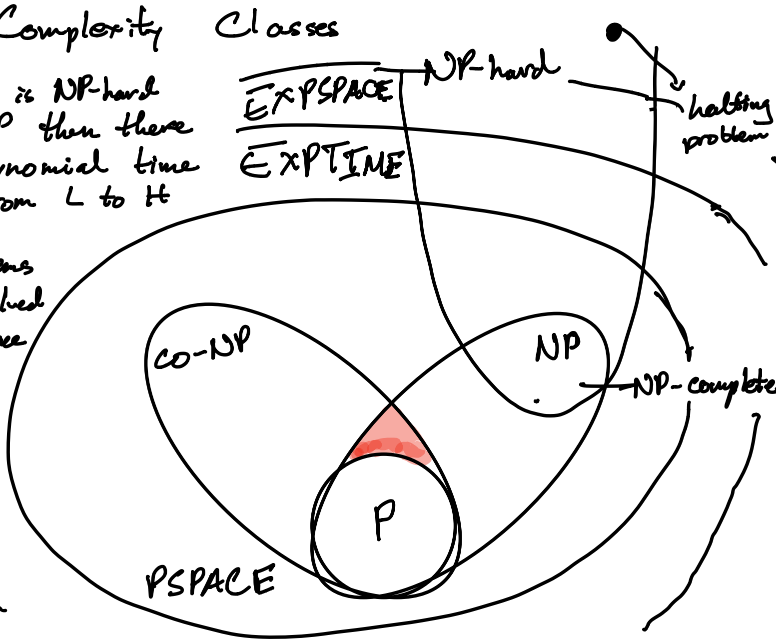


Re-review Complexity Classes

NP-hard: if H is NP-hard and $L \in NP$ then there exists a polynomial time reduction from L to H

PSPACE all problems that can be solved in polynomial space

EXPTIME problems that can be solved in exponential time



P: solvable by det. TM in poly time

NP: if answer is Yes then proof can be checked in poly time.

co-NP: if answer is No then there is a proof that can be checked in poly time

tautology: if for every assignment expr is true "x = y or x ≠ y"

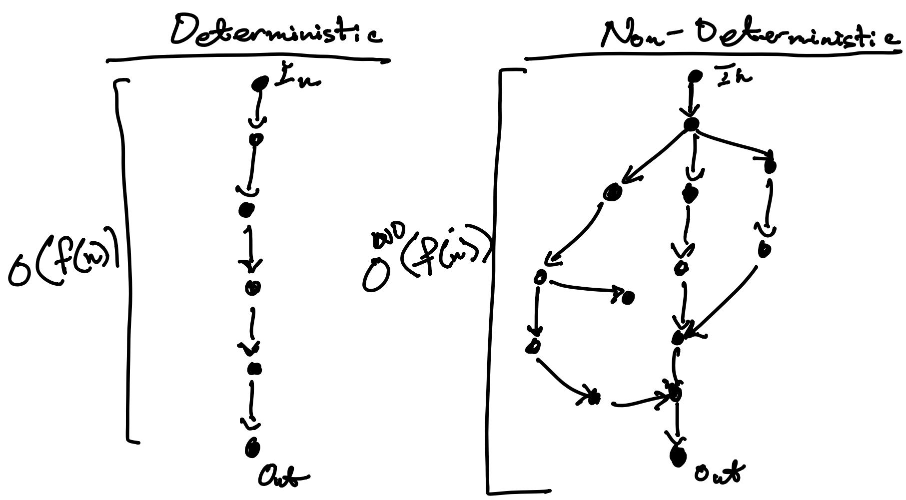
Factorization: does n have a factor smaller than k?

↳ in NP: "yes" can be checked by showing $n = d\left(\frac{n}{d}\right)$ where $d < k$

↳ in co-NP: "no" can be checked $n = x \cdot y \cdot z \cdot \ldots$ where all are Primes

P & NP:

NP: a set of decision problems that have a polynomial, non-deterministic algorithm



Analogous: DFA
NFA

Certifiers:

Certifiers are algorithms that verify a solution to a problem

A problem is in NP if it has a poly time certifier

Problem: X $\quad s \in X \quad t$

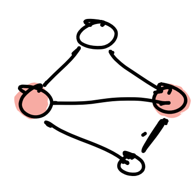
(3SAT) $\quad (\varphi) \quad$ (Assignment $= \{1, 0, 1; \ldots\}$
$\quad x_0\ x_1\ x_2 \ldots$

$C(s, t)$ output yes

An efficient certifier runs in polynomial time.

Example: Vertex Cover

Problem: Does G have a vertex cover of size $\leq k$



Certificate: $S \subseteq V$

Certifier $C(S, k)$ for every $e \in E$ checks to see one vertex is in S.

Example SAT:

Problem: Does φ has a sat assignment

Certificate: Assignment

Certifier: Check each clause and mark if clause is true
return yes if all clauses true