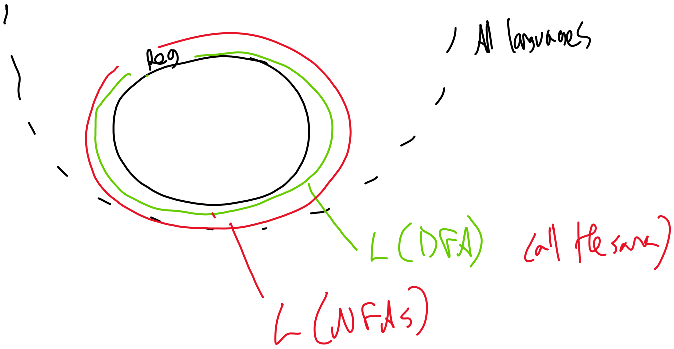
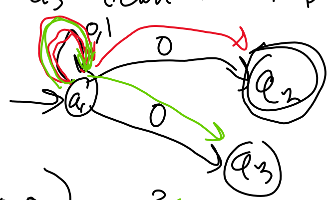
NFAs

Tuesday, September 1, 2020 9:30 AM



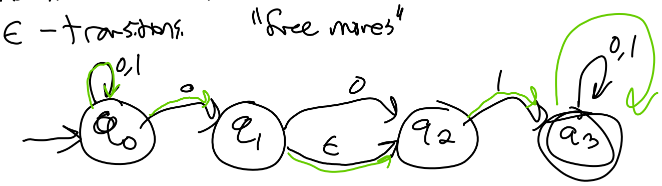
— NFAs as relaxation of DFAs.



$\delta^*(q_1, 100) = ? \{q_2, q_3, q_1\}$

Features of NFAs

- multiple transitions for same state and symbol
- No transitions is possible.
- ϵ - transitions. "free moves"

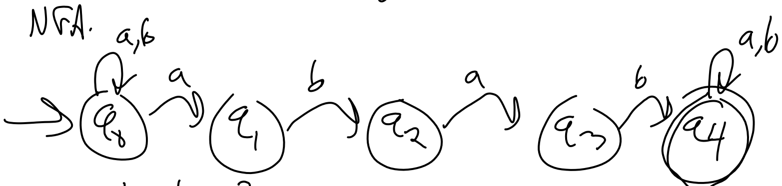


— NFA <u>accepts</u> string x if <u>some</u> path processing each symbol ends in A.

ex. 1011 ?

$q_0 \xrightarrow{1} q_0 \xrightarrow{0} q_1 \xrightarrow{\epsilon} q_2 \xrightarrow{1} q_3 \xrightarrow{1} q_3$ ✓

harder to show a string is NOT accepted by NFA.



ex ababa?

x ↓					
a					
b					
a					
b					
a					

ϵ reach ... $\bigcup$ ← accounting for multiple free moves.

ϵ reach (q) ...

δ^* for DFAs $\quad \delta^*: Q \times \Sigma^* \to Q$

δ^* for NFAs $\quad \delta^*: Q \times \Sigma^* \to \mathcal{P}(Q)$

Challenge in defining δ^* is accounting for free moves (ϵ-transitions)

- Can take ϵ-transitions before or after processing a symbol.
- Can take multiple ϵ-transitions in a row.

Closure operations.

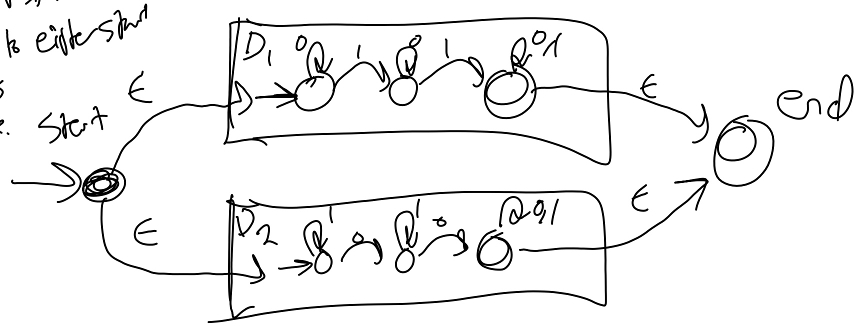
Given N s.t. $L(N) = L(D_1) \cup L(D_2)$ for DFAs D_1 and D_2.

ex. Contains at least two 1's or two 0's?

$(\Sigma^* 1 \Sigma^* 1 \Sigma^* + \Sigma^* 0 \Sigma^* 0 \Sigma^*)$

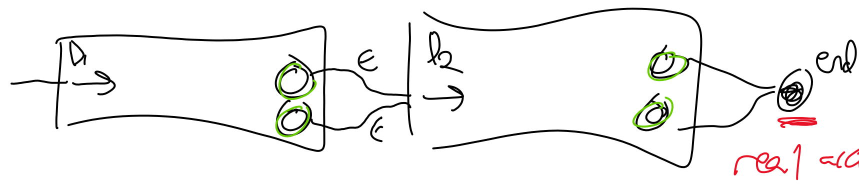
For $\cup$ closure

- add new start state
- add ϵ trans to either start
- link accept states to a new end state.



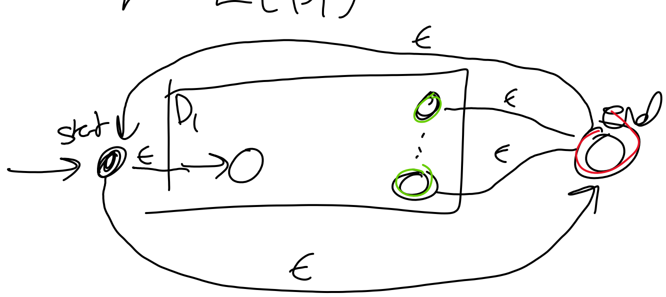
Concatenation: $\quad L = L(D_1) \circ L(D_2)$

Internal accepting states

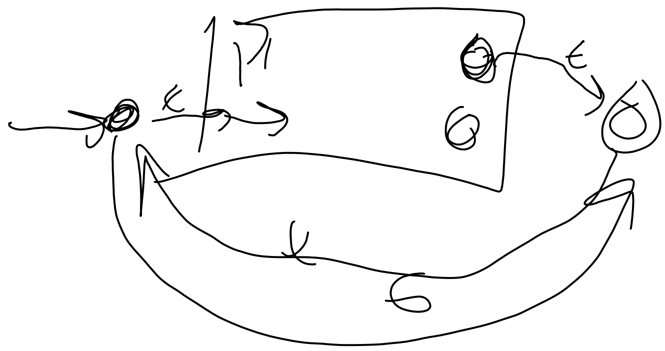


real accepting state.

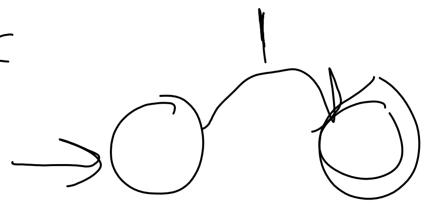
Kleene Star $\quad L = L(D_1)^*$



$L(D_1) = (10)^*$ $\quad \underline{1\,0}\ \cancel{1}$



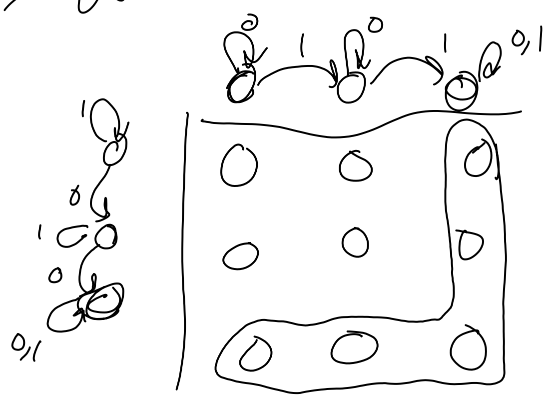
Ex: $N =$



is $111 \in L(N)$?

$L(N) = \{1\}$

Ex ≥ 2 1's OR ≥ 2 0's



$q_{i,j}$ means i 1's so far

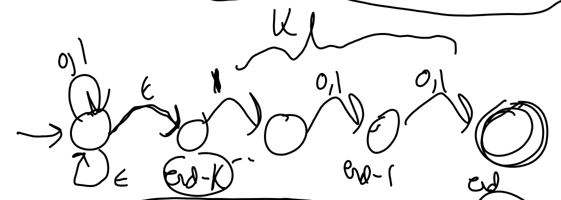
$A = \{a_1, a_2 \mid a_1 \in A_1 \text{ or } a_2 \in A_2\}$

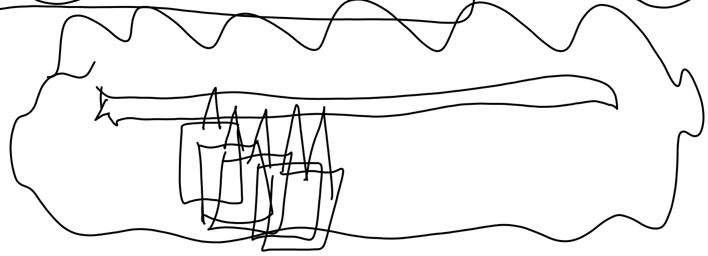
EXS of guessing.

– "guess" solutions to a problem and check them.

ex $L = \{ X \mid X \text{ has a "1" symbol } K \text{ symbols from the end} \}$

eg, $k=3$,





$Q = \{ \ldots \}$

alternative:

$$\underbrace{\Sigma^*}_{\text{anything}} \circ 1 \circ \underbrace{\Sigma^{k-1}}_{\text{any string of } k-1 \text{ symbols}}$$

Σ^* recognized by an NFA (Σ recognized by NFA, and NFA closed under $*$)

1 recognized by NFA

Σ^{k-1} recognized NFA (by $\circ$ closure $k-1$ times)

$\Sigma^* \circ 1 \circ \Sigma^{k-1}$ (by $\circ$ closure)

See slides for PREFIX

Ex. EveryOther

$$L' = \{ X \mid X = \underline{\text{everyother}}(w) \text{ for } \underline{\text{some}}\ w \in L \}$$

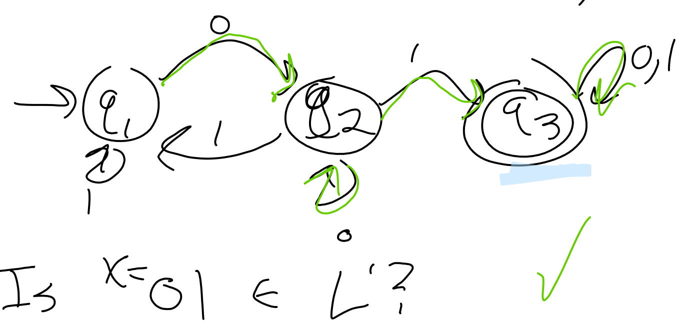
and L recognized by a DFA.

everyother(X) = { ε   ε = X

$$\text{everyother}(x) = \begin{cases} \ldots & \\ a & x = a \\ a \circ \text{everyother}(w) & x = abw \end{cases}$$

ex. everyother(10110110) = 1101

ex.



= D, $L = L(D)$

Is $x = 01 \in L'$?

$\Rightarrow$ 0 0 1 1 = w for some w, $w \in L$

e.g. 0011

$N' = (Q', \delta', A', s')$ given $D = (Q, \delta, A, s)$

$Q' = Q$

$s' = s$

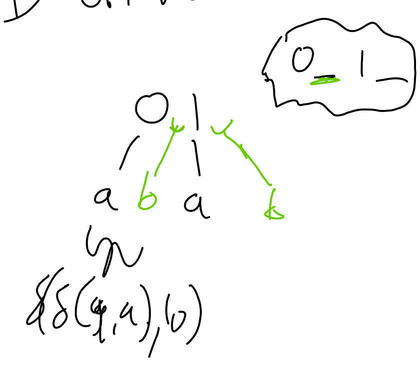
$\delta'(q, a) = \{ \delta(\delta(q,a), b) \mid b \in \Sigma \}$

Main idea:

read one symbol from x,

guess next symbol for w

Simulate D on w.



Thm: $L(N) = L'$

Proof: $\Rightarrow$ For every string $x \in L'$, N accepts x.

$\Leftarrow$ For every x accepted by N, $x \in L'$.
