CS/ECE 374 A ✧ Fall 2021

Conflict Midterm 2

November 9, 2021

Directions

- ***Don't panic!***
- If you brought anything except your writing implements, your hand-written double-sided 8½" × 11" cheat sheet, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- **We *strongly* recommend reading the entire exam before trying to solve anything.** If you think a question is unclear or ambiguous, please ask for clarification as soon as possible.
- The exam has five numbered questions, each worth 10 points. (Subproblems are not necessarily worth the same number of points.)
- Write your answers on blank white paper using a dark pen. Please start your solution to each numbered question on a new sheet of paper.
- You have **120 minutes** to write your solutions, after which you have 30 minutes to scan your solutions, convert your scan to a PDF file, and upload your PDF file to Gradescope.
- If you are ready to scan your solutions before 9:00pm, send a private message to the host of your Zoom call ("Ready to scan") and wait for confirmation before leaving the Zoom call.
- Gradescope will only accept PDF submissions. Please do not scan your cheat sheets or scratch paper. Please make sure your solution to each numbered problem starts on a new page of your PDF file. **Low-quality scans will be penalized.**
- Proofs are required for full credit if and only if we explicitly ask for them, using the word ***prove*** in bold italics.
- Finally, if something goes seriously wrong, send email to jeffe@illinois.edu as soon as possible explaining the situation. If you have already finished the exam but cannot submit to Gradescope for some reason, include a complete scan of your exam **as a PDF file** in your email. If you are in the middle of the exam, send Jeff email, continue working until the time limit, and then send a second email with your completed exam **as a PDF file**. Please do not email raw photos.

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1. Short answers:

   (a) Solve the recurrence $T(n) = 3T(n/2) + O(n^2)$.

   (b) Solve the recurrence $T(n) = 7T(n/2) + O(n^2)$.

   (c) Solve the recurrence $T(n) = 4T(n/2) + O(n^2)$.

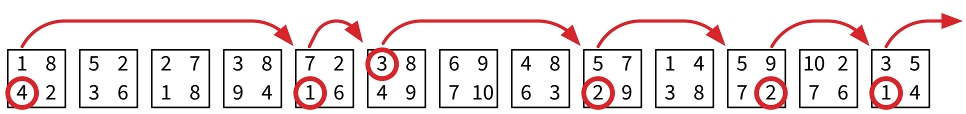
   (d) Draw a directed acyclic graph with at most ten vertices, exactly one source, exactly one sink, and more than one topological order.

   (e) Draw a directed graph with at most ten vertices, with distinct edge weights, that has more than one shortest path from some vertex s to some other vertex t.

   (f) Describe an appropriate memoization structure and evaluation order for the following (meaningless) recurrence, and give the running time of the resulting iterative algorithm to compute $Huh(1, n)$.

$$Huh(i,k) = \begin{cases} 0 & \text{if } i > n \text{ or } k < 0 \\ \min \begin{Bmatrix} Huh(i+1, k-2) \\ Huh(i+2, k-1) \end{Bmatrix} + A[i,k] & \text{if } A[i,k] \text{ is even} \\ \max \begin{Bmatrix} Huh(i+1, k-2) \\ Huh(i+2, k-1) \end{Bmatrix} - A[i,k] & \text{if } A[i,k] \text{ is odd} \end{cases}$$

2. *Quadhopper* is a solitaire game played on a row of n squares. Each square contains four positive integers. The player begins by placing a token on the leftmost square. On each move, the player chooses one of the numbers on the token's current square, and then moves the token that number of squares to the right. The game ends when the token moves past the rightmost square. The object of the game is to make as many moves as possible before the game ends.



A quadhopper puzzle that allows six moves. (This is ***not*** the longest legal sequence of moves.)

   (a) ***Prove*** that the obvious greedy strategy (always choose the smallest number) does not give the largest possible number of moves for every quadhopper puzzle.

   (b) Describe and analyze an efficient algorithm to find the largest possible number of legal moves for a given quadhopper puzzle.

3. Suppose you are given a directed graph $G = (V, E)$, each of whose vertices is either red, green, or blue. Edges in G do not have weights, and G is not necessarily a dag.

   Describe and analyze an algorithm to find a shortest path in G that contains at least one vertex of each color. (In particular, your algorithm must choose the best start and end vertices for the path.)

4. Your grandmother dies and leaves you her treasured collection of n radioactive Beanie Babies. Her will reveals that one of the Beanie Babies is a rare specimen worth 374 million dollars, but all the others are worthless. All of the Beanie Babies are equally radioactive, except for the valuable Beanie Baby, which is is either slightly more or slightly less radioactive, but you don't know which. Otherwise, as far as you can tell, the Beanie Babies are all identical.

   You have access to a state-of-the-art Radiation Comparator at your job. The Comparator has two chambers. You can place any two disjoint sets of Beanie Babies in Comparator's two chambers; the Comparator will then indicate which subset emits more radiation, or that the two subsets are equally radioactive. (Two subsets are equally radioactive if and only if they contain the same number of Beanie Babies, and they are all worthless.) The Comparator is slow and consumes a *lot* of power, and you really aren't supposed to use it for personal projects, so you *really* want to use it as few times as possible.

   Describe an efficient algorithm to identify the valuable Beanie Baby. How many times does your algorithm use the Comparator in the worst case, as a function of n?

5. Ronnie and Hyde are a professional robber duo who plan to rob a house in the Leverwood neighborhood of Sham-Poobanana. They have managed to obtain a map of the neighborhood in the form of a directed graph G, whose vertices represent houses, whose edges represent one-way streets.

   - One vertex s represents the house that Ronnie and Hyde plan to rob.
   - A set X of special vertices designate eXits from the neighborhood.
   - Each directed edge $u \to v$ has a non-negative weight $w(u \to v)$, indicating the time required to drive directly from house u to house v.
   - Thanks to Leverwood's extensive network of traffic cameras, speeding or driving backwards along any one-way street would mean certain capture.

   Describe and analyze an algorithm to compute the shortest time needed to exit the neighborhood, starting at house s. The input to your algorithm is the directed graph $G = (V, E)$, with clearly marked subset of exit vertices $X \subseteq V$, and non-negative weights $w(u \to v)$ for every edge $u \to v$.