Nancy Gunter, the founding dean of the new Parisa Tabriz School of Computer Science, has commissioned a series of snow ramps on the south slope of the Orchard Downs sledding hill[1] and challenged Erhan Hajek, head of the Department of Electrical and Computer Engineering, to a sledding contest. Erhan and Nancy will both sled down the hill, each trying to maximize their air time. The winner gets to expand their department/school into Siebel Center, the ECE Building, *and* the new Campus Instructional Facility; the loser has to move their entire department/school under the Boneyard bridge behind Everitt Lab.

Whenever Nancy or Erhan reaches a ramp *while on the ground*, they can either use that ramp to jump through the air, possibly flying over one or more ramps, or sled past that ramp and stay on the ground. Obviously, if someone flies over a ramp, they cannot use that ramp to extend their jump.

1. Suppose you are given a pair of arrays $Ramp[1..n]$ and $Length[1..n]$, where $Ramp[i]$ is the distance from the top of the hill to the ith ramp, and $Length[i]$ is the distance that any sledder who takes the ith ramp will travel through the air.

   Describe and analyze an algorithm to determine the maximum *total* distance that Erhan or Nancy can travel through the air.

2. Uh-oh. The university lawyers heard about Nancy and Erhan's little bet and immediately objected. To protect the university from both lawsuits and sky-rocketing insurance rates, they impose an upper bound on the number of jumps that either sledder can take.

   Describe and analyze an algorithm to determine the maximum total distance that Nancy or Erhan can spend in the air *with at most k jumps*, given the original arrays $Ramp[1..n]$ and $Length[1..n]$ and the integer k as input.

Harder problem to think about later:

3. When the lawyers realized that imposing their restriction didn't immediately shut down the contest, they added yet another restriction: No ramp may be used more than once! Disgusted by all the legal interference, Erhan and Nancy give up on their bet and decide to cooperate to put on a good show for the spectators.

   Describe and analyze an algorithm to determine the maximum total distance that Nancy and Erhan can spend in the air, each taking at most k jumps (so at most $2k$ jumps total), and with each ramp used at most once.

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[1]The north slope is faster, but too short for an interesting contest.