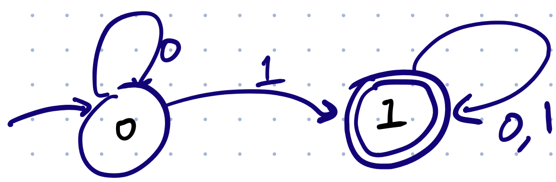
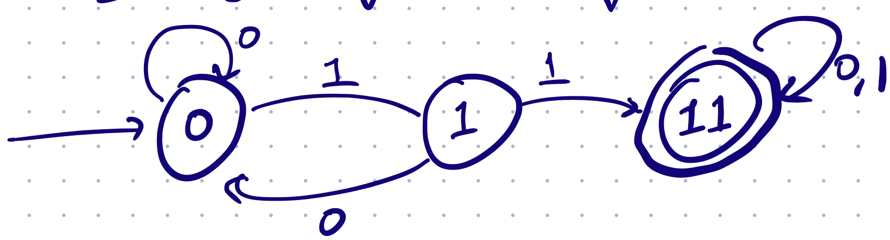
LECTURE - 5

TODAY : FOOLING SETS,
PROVING NON REGULARITY,
NFAs

$L_1 = \{$ strings containing $1 \}$



$L_2 = \{$ strings containing $11 \}$



Find 3 strings w, x, y s.t.

$$\delta^*(s,w) \neq \delta^*(s,x)$$
$$\delta^*(s,x) \neq \delta^*(s,y)$$
$$\delta^*(s,w) \neq \delta^*(s,y)$$

00, 01, 11.

Suppose $\underbrace{\delta^*(s,00) = \delta^*(s,01)}_{=p} \Rightarrow \boxed{\delta^*(s,001) = \delta^*(s,011)}$

$$\delta^*(s,001) = \delta(\delta^*(s,00),1) = \delta(p,1)$$

But $001 \notin L \Rightarrow \delta^*(s, 001) \notin A$

$011 \in L \Rightarrow \delta^*(s, 011) \in A$

So $\delta^*(s, 001)$ cannot equal $\delta^*(s, 011)$.

We have a contradiction.

Therefore, our assumption must be false.

$$\delta^*(s, 00) \neq \delta^*(s, 01).$$

FOOLING SET:

Set S of strings such that for any two strings $x, y \in S$, $\exists$ string z such that

$$(xz \in L) \text{ XOR } (yz \in L).$$

Alternatively, such that

$$(xz \in L \text{ and } yz \notin L) \text{ OR } (xz \notin L \text{ and } yz \in L).$$

Claim: Let $L = \{\text{strings containing } 11\}$

Then $F = \{00, 01, 11\}$ is a fooling set for L.

Proof: We must show that for every $x, y \in F$, $\exists\, z \in \Sigma^*$ s.t.

distinguishing suffix

$$xz \in L \quad \text{XOR} \quad yz \in L.$$

1) $x = 00$, $y = 01$. Then $001 \notin L$, $011 \in L$

$\Rightarrow 1$ is a distinguishing suffix.

2) $x = 01$, $y = 11$. Then $010 \notin L$, $110 \in L$.

$\Rightarrow$ 0 is a distinguishing suffix.

3) $x = 00$, $y = 11$. Then $000 \notin L$, $110 \in L$

$\Rightarrow$ 0 is a distinguishing suffix.

Myhill-Nerode Theorem (partial)

Min # states in DFA for L

= Max of the size of a fooling set for L.

If a language L has an infinite fooling set, then L is NOT regular.

PROVING THAT A LANGUAGE IS NOT REGULAR

$L = \{ 0^n 1^n \mid n \geq 0 \} = \{ \varepsilon, 01, 0011, 000111, \ldots \}$

Fooling set S for L :

$\forall\, x, y \in S$, $\exists$ distinguishing suffix z.

Let $F = \{ \varepsilon, 0, 00, 000 \ldots \}$

$= 0^*$.

Let x and y be arbitrary strings in F.

Then $x = 0^i$, $y = 0^j$ for $i, j \geq 0$ and $i \neq j$.

Let $z = 1^i$.

Then $xz = 0^i 1^i \in L$.

$yz = 0^j 1^i$ for $j \neq i \Rightarrow yz \notin L$.

So, z is a distinguishing suffix for x, y.

So, F is a fooling set for L.

Because F is infinite, L <u>cannot</u> be regular.

<u>Claim</u> : L_4 = palindromes over $\Sigma = \{0, 1\}$

$= \{ w \mid w = w^R \}$

$= \{ \varepsilon, 0, 1, 00, 11, 010 \ldots \}$

not 01 or 10 or 001 . . . .

Let $F = \{\varepsilon, 0, 00, 000 \ldots\} = 0^*$.

Let x and y be any two strings in F.

$\Rightarrow$ $x = 0^i$, $y = 0^j$ $\quad i, j \geq 0, \; i \neq j$.

Let $z = 1^i 0^i$.

$xz = 0^i 1^i 0^i$ $\quad (xz)^R = z^R \cdot x^R$

$= (1^i 0^i)^R \cdot (0^i)^R$

$= (0^i)^R \cdot (1^i)^R \cdot (0^i)^R$

$= 0^i 1^i 0^i = xz$.

$yz = 0^j 1^i 0^i \quad (yz)^R = z^R \cdot y^R$

$= (1^i 0^i)^R \cdot (0^j)^R$

$= 0^i 1^i 0^j \neq yz.$

Another distinguishing suffix is $z = 10^i$ (HW).

<u>Claim.</u> $L_5 = \{ www \mid w \in \Sigma^* \}$

$= \{ \varepsilon, 000, 111, 010101 \ldots \}$

Let $F = 0^*$.

Let x, y be arbitrary 2 strings in F.

Then $x = 0^i$, $y = 0^j$ for $i, j \geq 0$, $i \neq j$.

Let $z = 10^i 10^i 1$

$xz = 0^i 10^i 10^i 1 = www$ for $w = 0^i 1 \in L_5$

$yz = \underline{0^j 1} 0^i 1 0^i 1 \neq www$ for any w.

$\notin L_5$

z is distinguishing suffix for x and y.

F is a fooling set for L_5.

Because F is infinite, L_5 <u>cannot</u> be regular.

$x = 0^i \quad , \; y = 0^j \quad , \; i \neq j$

$z = 0^{2i}$

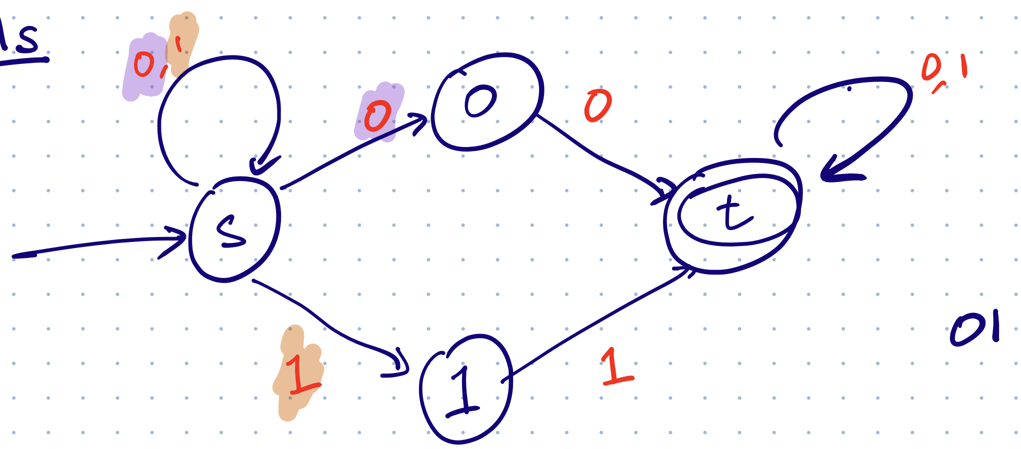
$xz = 0^{3i} = 0^i\,0^i\,0^i = www. \in L$

$yz = 0^{j+2i}$ But if $j = 4i$, $yz = 0^{6i}$
$= 0^{2i}\,0^{2i}\,0^{2i} \in L$

Really reasoning about $L_5 \cap 0^*10^*10^*1$.

(Non-Deterministic Finite-state Automata)

NFAs



$NFA = (\Sigma, Q, s, A, \delta)$

Q - Finite states

s - start state

$A \subseteq Q$ accepting states

$\delta : Q \times \Sigma \rightarrow \cancel{Q}\ 2^Q$
(subsets of Q).

NFA accepts $w = 01001\ldots$ iff there is <u>any</u> sequence of transitions.

$$s \xrightarrow{0} q_1 \xrightarrow{1} q_2 \cdots\cdots \longrightarrow q_n \in A.$$

(On the other hand, DFA accepts if <u>the</u> sequence of transitions.

$$s \xrightarrow{0} q_1 \xrightarrow{1} q_2 \cdots\cdots q_n \in A.$$)