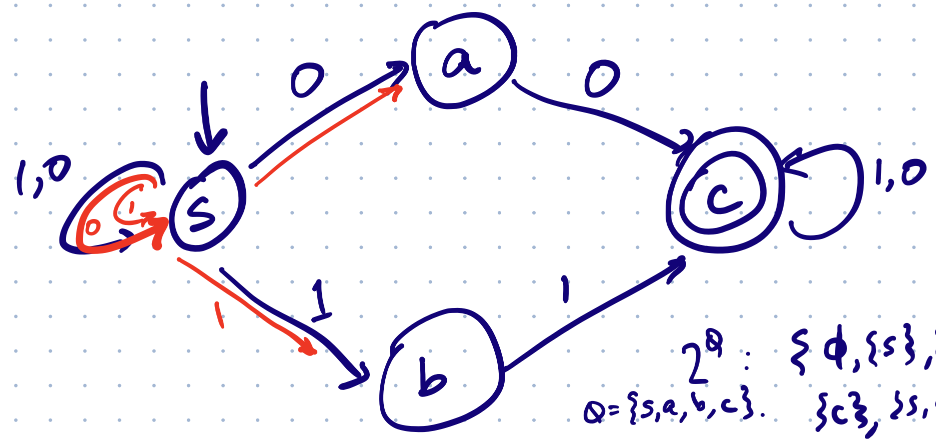
LECTURE - 6

NONDETERMINISTIC FINITE AUTOMATA



2^Q : $\{\phi, \{s\}, \{a\}, \{b\}, \{c\}, \{s,a\}, \{s,b\} \dots\}$

$Q = \{s, a, b, c\}$.

NFA accepts if $\exists$ a walk $s \xrightarrow{w_1} q_1 \xrightarrow{w_2} \dots\dots \rightarrow q_n$

"there exists" $\qquad q_n \in A$

$\delta(s, 0) = \{s, a\}$

NONDETERMINISM

- Magic oracles
- Parallel worlds / threads
- Verification

$$\delta : Q \times \Sigma \rightarrow 2^Q$$

Power set of Q

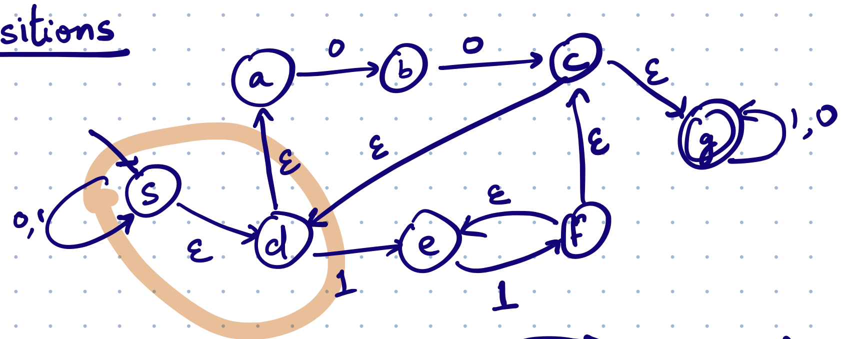
set of all subsets of Q

$$(q, a) \rightarrow \{q_1, q_2, q_3\}$$

Kleene's Thm.

$$\begin{tikzcd} \text{DFA} \arrow[r, bend left, "\text{trivial}"] & \text{NFA} \arrow[l, bend left, "\text{subset constructions}"] \arrow[r, bend left, "\text{dynamic programming}"] & \text{reg. expressions} \arrow[l, bend left, "\text{parse}"] \end{tikzcd}$$

ε-transitions



NFA with ε-transitions $\underset{\text{trivial}}{\overset{\text{non-trivial}}{\rightleftarrows}}$ NFA without ε-transitions

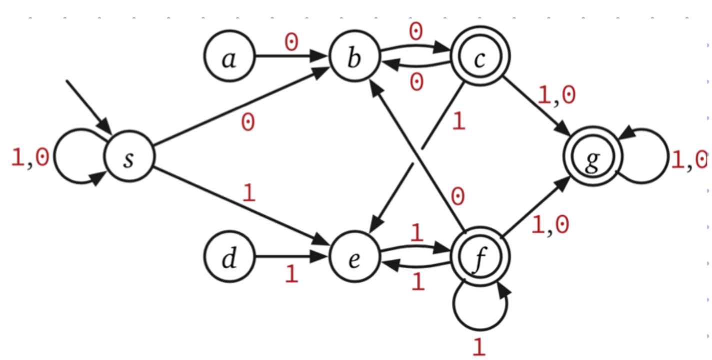
$\varepsilon\text{-reach}(q) = \{$all states reachable from q with ε-transitions$\}$.

$\varepsilon\text{-reach}(s) = \{s, d, a\}$

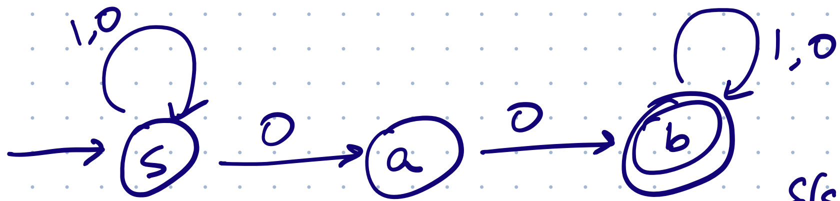
Let $\{\Sigma, Q, s, A, \delta)$ be any ε-NFA.

Then $\{\Sigma, Q', s', A', \delta')$ is an NFA without ε transitions

where
$$Q' = Q$$
$$s' = s$$
$$A' = \{q \in Q \text{ s.t. } \varepsilon\text{-reach}(q) \cap A \neq \phi\}$$
$$\delta'(q, a) = \delta(\varepsilon\text{-reach}(q), a)$$
$$= \bigcup_{p \in \varepsilon\text{-reach}(q)} \delta(p, a)$$



Convert NFA to a DFA.



$\delta(s,0) = \{s, a\}$

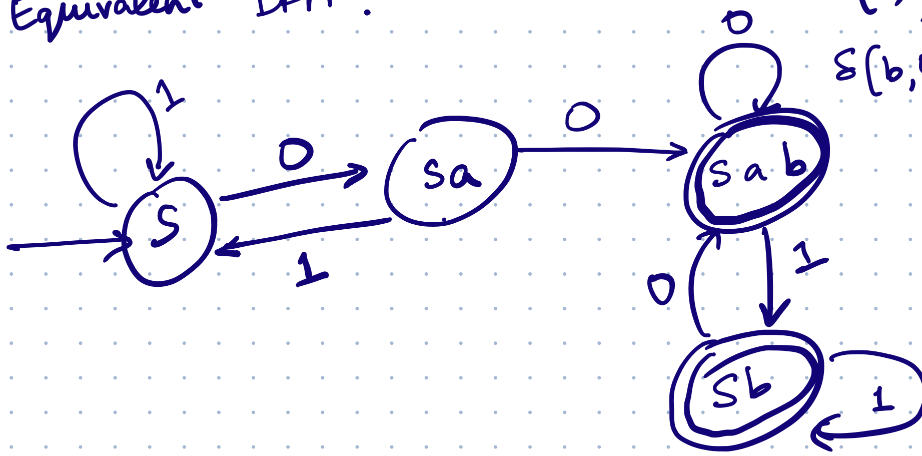
$\cup$

$\delta(a,0) = \{b\}$

$\cup$

$\delta(b,0) = \{b\}$

Equivalent DFA.



$$\boxed{\text{SUBSET CONSTRUCTION : NFA} \rightarrow \text{DFA}}$$

For every

NFA $N = (\Sigma, Q, s, A, \delta)$ $\quad \delta : Q \times \Sigma \rightarrow 2^Q$

$\exists$ DFA $M = (\Sigma, Q', s', A', \delta')$ $\quad \delta' : Q' \times \Sigma \rightarrow Q'$

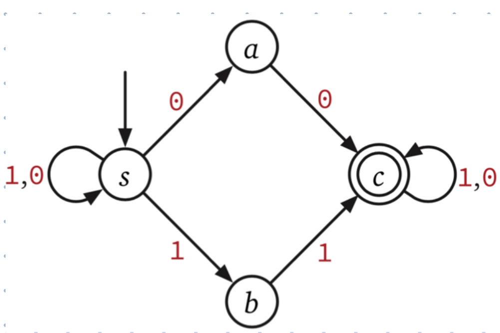
where set of all states of DFA $\leftarrow$ $Q' = 2^Q$ $\leftarrow$ set of all subsets of states of NFA.

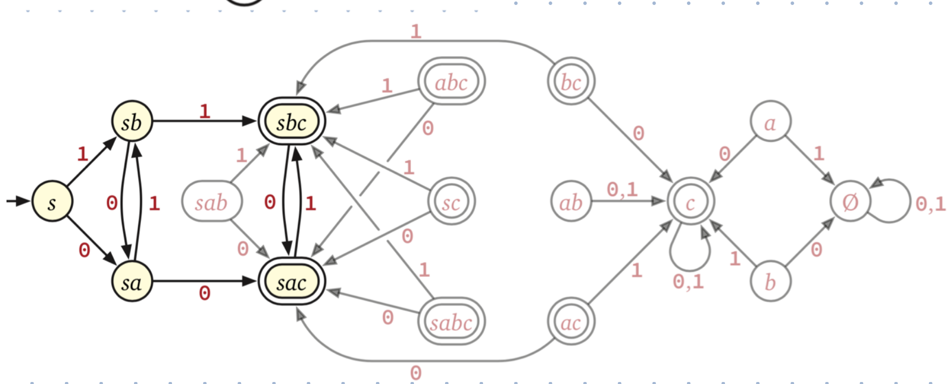
$$s' = \{s\}$$

$$A' = \{ s \subseteq Q \mid s \cap A \neq \emptyset \}$$

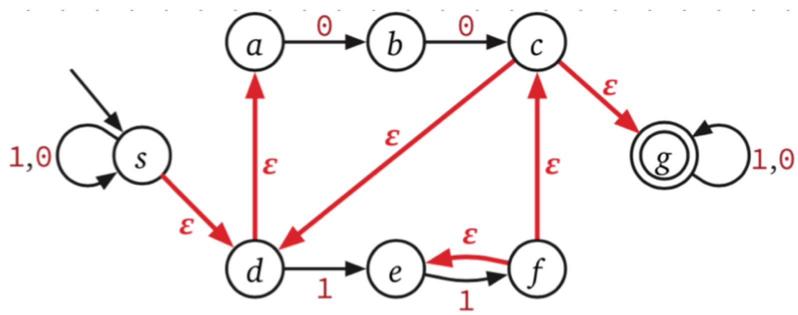
$$\delta'(p, a) = \bigcup_{q \in p} \delta(q, a)$$

Here's what happens when you apply the subset construction.





INCREMENTAL SUBSET CONSTRUCTION.



states	ε-reach	Acc?	δ(–,0)	δ(–,1)
s	sda	×	sb	se
sb	sdab	×	sbc	se
se				
sbc				

q'	ε-reach(q')	$q' \in A'$?	$\delta'(q', 0)$	$\delta'(q', 1)$
s	*sad*		*sb*	*se*
sb	*sabd*		*sbc*	*se*
se	*sade*		*sb*	*sef*
sbc	*sabcdg*	✓	*sbcg*	*seg*
sef	*sacdefg*	✓	*sbg*	*sefg*
sbcg	*sabcdg*	✓	*sbcg*	*seg*
seg	*sadeg*	✓	*sbg*	*sefg*
sbg	*sabdg*	✓	*sbcg*	*seg*
sefg	*sacdefg*	✓	*sbg*	*sefg*