CS 374 Lab 1: Preliminaries, Strings, and Languages

Date: January 20, 2016.

Recall the following notions: Sets, set building notation, subset, proper subset, empty set, Venn diagram, Cartesian product of sets, power set of a set.

Problem 1. [Category: Comprehension] For each of the following statements answer **True**, **False**, or **Meaningless**.

- $\{a, b, c\} \cap \{d, e\} = \emptyset$
- $\{a, b, c\} \cap \{d, e\} = \{\emptyset\}$
- $\{a, b, c\} \cup \{d, a, e\} = \{a, b, c, d, a, e\}$
- $\emptyset \in \{\emptyset, a\}$
- $S \in \mathcal{P}(S)$, where S is a set and $\mathcal{P}(S)$ is the powerset of S
- $a \in \mathcal{P}(\{a\})$
- $\{a, b\} + \{c, d\} = \{a, b, c, d\}$
- $\{\{a, a\}\} = \{a, a\}$
- $\{\{a\}, \{a\}\} = \{a, a\}$
- $\{a, b\} \times \{b\} = \{(a, b), (b, b)\}$
- $\{a, b\} \times \{c, d\} = \{c, d\} \times \{a, b\}$

Problem 2. [Category: Comprehension+Proof] Let us define a set U_n inductively as follows.

- $U_1 = \{1\}$
- $U_i = U_{i-1} \cup \{\max(U_{i-1}) + 2(i-1) + 1\}$

Answer the following questions about the set U_n.

1. What is U_2? What is U_3?
2. Is $U_{n-1} \in U_n$?
3. What is U_n? Prove your answer.

Problem 3. [Category: Comprehension+Proof] For a string $w \in \{0, 1\}^*$, w^c is inductively defined as follows.

$$w^c = \begin{cases} \epsilon & \text{if } w = \epsilon \\ 1 & \text{if } w = 0 \\ 0 & \text{if } w = 1 \\ (a^c)(u^c) & \text{if } w = au \text{ where } a \in \{0, 1\},\ u \in \{0, 1\}^* \end{cases}$$

1. What is $(10101)^c$?
2. Prove that for any strings $u, v \in \{0,1\}^*$, $u^c \bullet v^c = (u \bullet v)^c$.
3. Recall w^R denotes the reverse of string w defined as

$$w^R = \begin{cases} \epsilon & \text{if } w = \epsilon \\ (u^R) \bullet a & \text{if } w = a \bullet u \text{ where } a \in \Sigma,\ u \in \Sigma^* \end{cases}$$

   Prove that $(w^c)^R = (w^R)^c$.