CS/ECE 374 ✧ Spring 2017

Homework 9

Due Wednesday, April 19, 2017 at 10am

Groups of up to three people can submit joint solutions. Each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the Gradescope names and email addresses of each group member. In addition, whoever submits the homework must tell Gradescope who their other group members are.

The following unnumbered problems are not for submission or grading. No solutions for them will be provided but you can discuss them on Piazza.

- Let $R_1, R_2, \ldots, R_n$ be a set of red intervals each of which is specified by its two end points. Let $B_1, B_2, \ldots, B_m$ be a set of blue intervals each of which is also specified by its two end points. You wish to find the smallest number of blue intervals that *cover* the red intervals. A blue interval B_j covers a red interval R_i if they contain the same point p on the line. All intervals are close in the sense that the end points are contained in the interval. Describe a greedy algorithm for this problem and prove its correctness.

- We saw in lecture that Borouvka's algorithm for MST can be implemented in $O(m \log n)$ time where m is the number of edges and n is the number of nodes. We also saw that Prim's algorithm can be implemented in $O(m + n \log n)$ time. Obtain an algorithm for MST with running time $O(m \log \log n)$ by running Borouvka's algorithm for some number of steps and then switching to Prim's algorithm. This algorithm is better than either of the algorithms when $m = \Theta(n)$. Formalize the algorithm, specify the parameters and argue carefully about the implementation and running time details. Briefly justify the correctness of the algorithm assuming that the edge weights are unique.

1. Red street in the city Shampoo-Banana can be modeled as a straight line starting at 0. The street has n houses at locations $x_1, x_2, \ldots, x_n$ on the line. The local cable company wants to install some new fiber optic equipment at several locations such that every house is within distance r from one of the equipment locations. The city has granted permits to install the equipment, but only at some m locations on the street given y locations $y_1, y_2, \ldots, y_m$. For simplicity assume that all the x and y values are distinct. You can also assume that $x_1 < x_2 < \ldots < x_n$ and that $y_1 < y_2 < \ldots < y_m$.

   - Describe a greedy algorithm that finds the minimum number of equipment locations that the cable company can build to satisfy the desired constraint that every house is within distance r from one of them. Your algorithm has to detect if a feasible solution does not exist. Prove the correctness of the algorithm. One way to do this by arguing that there is an optimum solution that agrees with the first choice of your greedy algorithm.

- **Not to submit:** The cable company has realized subsequently that not all locations are equal in terms of the cost of installing equipment. Asssume that c_j is the cost at location y_j. Describe a dynamic programming algorithm that minimizes the total cost of installing equipment under the same constraint as before. Do you see why a greedy algorithm may not work for this cost version?

2. Let $G = (V, E)$ be an edge-weighted undirected graph. We are interested in computing a minimum spanning tree T of G to find a cheapest subgraph that ensures connectivity. However, some of the nodes in G are unrealiable and may fail. If a node fails it can disconnect the tree T unless it is a leaf. Thus, you want to find a cheapest spanning tree in G in which all the unreliable nodes (which is a given subset $U \subset V$) are leaves. Describe an efficient for this problem. Note that your algorithm should also check wither a feasible spanning tree satisfying the given constraint exists in G.

3. Consider the language $L_{\text{OH}} = \{\langle M \rangle \mid M \text{ halts on at least one input string}\}$. Note that for $\langle M \rangle \in L_{\text{OH}}$, it is not necessary for M to *accept* any string; it is sufficient for it to *halt* on (and possibly rejects) some string. Prove that L_{OH} is undecidable.