

Prove that each of the following languages is *not* regular.

1.  $\{0^{2^n} \mid n \geq 0\}$
2.  $\{0^{2^n}1^n \mid n \geq 0\}$
3.  $\{0^m1^n \mid m \neq 2n\}$
4. Strings over  $\{0, 1\}$  where the number of 0s is exactly twice the number of 1s.
5. Strings of properly nested parentheses  $()$ , brackets  $[]$ , and braces  $\{\}$ . For example, the string  $([])\{\}$  is in this language, but the string  $([]) ]$  is not, because the left and right delimiters don't match.
6. Strings of the form  $w_1\#w_2\#\dots\#w_n$  for some  $n \geq 2$ , where each substring  $w_i$  is a string in  $\{0, 1\}^*$ , and some pair of substrings  $w_i$  and  $w_j$  are equal.

**Work on these later:**

7.  $\{0^{n^2} \mid n \geq 0\}$
8.  $\{w \in (0 + 1)^* \mid w \text{ is the binary representation of a perfect square}\}$