

1. A **basic arithmetic expression** is composed of characters from the set  $\{1, +, \times\}$  and parentheses. Almost every integer can be represented by more than one basic arithmetic expression. For example, all of the following basic arithmetic expressions represent the integer 14:

$$\begin{aligned}
 &1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 \\
 &((1 + 1) \times (1 + 1 + 1 + 1 + 1)) + ((1 + 1) \times (1 + 1)) \\
 &(1 + 1) \times (1 + 1 + 1 + 1 + 1 + 1 + 1) \\
 &(1 + 1) \times (((1 + 1 + 1) \times (1 + 1)) + 1)
 \end{aligned}$$

Describe and analyze an algorithm to compute, given an integer  $n$  as input, the minimum number of 1's in a basic arithmetic expression whose value is equal to  $n$ . The number of parentheses doesn't matter, just the number of 1's. For example, when  $n = 14$ , your algorithm should return 8, for the final expression above. The running time of your algorithm should be bounded by a small polynomial function of  $n$ .

2. **To think about later:** Suppose you are given a sequence of integers separated by  $+$  and  $-$  signs; for example:

$$1 + 3 - 2 - 5 + 1 - 6 + 7$$

You can change the value of this expression by adding parentheses in different places. For example:

$$\begin{aligned}
 &1 + 3 - 2 - 5 + 1 - 6 + 7 = -1 \\
 &(1 + 3 - (2 - 5)) + (1 - 6) + 7 = 9 \\
 &(1 + (3 - 2)) - (5 + 1) - (6 + 7) = -17
 \end{aligned}$$

Describe and analyze an algorithm to compute, given a list of integers separated by  $+$  and  $-$  signs, the maximum possible value the expression can take by adding parentheses. Parentheses must be used only to group additions and subtractions; in particular, do not use them to create implicit multiplication as in  $1 + 3(-2)(-5) + 1 - 6 + 7 = 33$ .