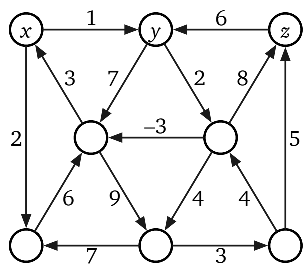
Write your answers in the separate answer booklet.
Please return this question sheet and your cheat sheet with your answers.

1. ***Clearly*** indicate the following structures in the directed graph below, or write NONE if the indicated structure does not exist. Don't be subtle; to indicate a collection of edges, draw a heavy black line along the entire length of each edge.



   (a) A depth-first tree rooted at x.
   (b) A breadth-first tree rooted at y.
   (c) A shortest-path tree rooted at z.
   (d) The shortest directed cycle.

2. After a few weeks of following your uphill-downhill walking path to work, your boss demands that you start showing up to work on time, so you decide to change your walking strategy. Your new goal is to walk to the highest altitude you can (to maximize exercise), while keeping the total length of your walk from home to work below some threshold (to make sure you get to work on time). Describe and analyze an algorithm to compute your new favorite route.

   Your input consists of an ***undirected*** graph G, where each vertex v has a height $h(v)$ and each edge e has a positive length $\ell(e)$, along with a start vertex s, a target vertex t, and a maximum length L. Your algorithm should return the ***maximum height*** reachable by a walk from s to t in G, whose total length is at most L.

   [Hint: This is the same input as HW8 problem 1, but the problem is completely different. In particular, the number of uphill/downhill switches in your walk is irrelevant.]

3. Suppose you have an integer array $A[1..n]$ that *used* to be sorted, but Swedish hackers have overwritten k entries of A with random numbers. Because you carefully monitor your system for intrusions, you know *how many* entries of A are corrupted, but not *which* entries or what the values are.

   Describe an algorithm to determine whether your corrupted array A contains an integer x. Your input consists of the array A, the integer k, and the target integer x. For example, if A is the following array, $k = 4$, and $x = 17$, your algorithm should return True. (The corrupted entries of the array are shaded.)

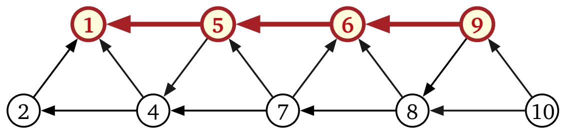
2	3	99	7	11	13	17	19	25	29	31	−5	41	43	47	53	8	61	67	71

   Assume that x is not equal to any of the the corrupted values, and that all n array entries are distinct. Report the running time of your algorithm as a function of n and k. A solution only for the special case $k = 1$ is worth 5 points; a complete solution for arbitrary k is worth 10 points. *[Hint: First consider $k = 0$; then consider $k = 1$.]*

4. Let G be a ***directed*** graph, where every vertex v has an associated height $h(v)$, and for every edge $u\rightarrow v$ we have the inequality $h(u) > h(v)$. Assume all heights are distinct. The *span* of a path from u to v is the height difference $h(u) - h(v)$.

   Describe and analyze an algorithm to find the ***maximum span*** of a path in G with at most k edges. Your input consists of the graph G, the vertex heights $h(\cdot)$, and the integer k. Report the running time of your algorithm as a function of V, E, and k.

   For example, given the following labeled graph and the integer $k = 3$ as input, your algorithm should return the integer 8, which is the span of the downward path $9\rightarrow 6\rightarrow 5\rightarrow 1$.



   [Hint: This is a very different question from problem 2.]

5. Suppose you are given a directed graph G where some edges are red and the remaining edges are blue, along with two vertices s and t. Describe an algorithm to compute the length of the shortest walk in G from s to t that traverses an even number of red edges and an even number of blue edges. If the walk traverses the same edge multiple times, each traversal counts toward the total for that color.

   For example, if you are given the graph below (where single arrows are red and double arrows are blue), your algorithm should return the integer 6, because the shortest legal walk from s to t is $s\rightarrow a\rightarrow b\Rightarrow d\Rightarrow a\rightarrow b\rightarrow t$.