Why are we here?

Theoretical computer science

What can be computed?
...quickly? How?
...under resource constraints?
or can't?

Computers are stupid.
People are clever. Clever is bad.

Today: Strings

Definition: A string is either
- nothing
- (a, x) where $a \in \Sigma$ "alphabet" "symbol"
  where x is a string.

"empty string" ε

$a \cdot x$ ax

STRING

$(S, (T, (R, (I, (N, (G, \varepsilon))))))$

set of all strings over Σ $\longrightarrow$ $\boxed{\Sigma^*}$

Length – "# of symbols"

$$|w| = \begin{cases} 0 & \text{if } w = \varepsilon \\ 1 + |x| & \text{if } w = ax \end{cases}$$

Concatenation – "write one after the other"

(F, OOT) • BALL = (F, OOTBALL)

$$w \cdot x = \begin{cases} x & \text{if } w = \varepsilon \\ a \cdot (y \cdot x) & \text{if } w = ay \end{cases}$$

$\varepsilon \cdot \varepsilon = \varepsilon$

$\varepsilon \cdot$ BALL BALL

FOOT $\cdot\, \varepsilon$ FOOT

$|w \cdot x| = |w| + |x|$ for all strings w and x

Proof:

Let w and x be arbitrary strings.

Assume for all strings y shorter than w that $|y \cdot x| \leq |y| + |x|$

There are two cases:

- If $w = \varepsilon$

$|w \cdot x| = |\varepsilon \cdot x|$

$= |x|$ [by definition of $\cdot$]

$= 0 + |x|$ [duh]

$= |\varepsilon| + |x| = |w| + |x|$ [by def. $|\varepsilon|$]

• If $w = ay$

$$
\begin{aligned}
|w \cdot x| &= |(ay) \cdot x| \\
&= |a \cdot (y \cdot x)| && [\text{by def. of } \cdot] \\
&= 1 + |y \cdot x| && [\text{by def of } |\,|] \\
&= 1 + |y| + |x| && [\text{by IH}] \\
&= |ay| + |x| = |w| + |x| && [\text{by def of } |\,|]
\end{aligned}
$$

Therefore $|w \cdot x| = |w| + |x|$