CS/ECE 374 A ✧ Spring 2018

Fake Midterm 1

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Real name:	Jeff Erickson
NetID:	jeffe

Gradescope name:	Flash
Gradescope email:	flash@gordon.mil

- ***Don't panic!***
- If you brought anything except your writing implements, your **hand-written** double-sided 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
- Please clearly print your real name, your university NetID, your Gradescope name, and your Gradescope email address in the boxes above. However, if you are using your real name and your university email address on Gradescope, you do ***not*** need to write everything twice.
- Please also print **only the name you are using on Gradescope** at the top of every page of the answer booklet. These are the pages we will scan into Gradescope.
- Please do not write outside the black boxes on each page; these indicate the area of the page that the scanner can actually see.
- If you run out of space for an answer, feel free to use the scratch pages at the back of the answer booklet, but **please clearly indicate where we should look.**
- Proofs are required for full credit if and only if we explicitly ask for them, using the word ***prove*** in bold italics.
- Please return ***all*** paper with your answer booklet: your question sheet, your cheat sheet, and all scratch paper.

CS/ECE 374 A ✧ Spring 2018 **Fake Midterm 1 Problem 1**	Gradescope name: Flash

For each statement below, check "True" if the statement is ***always*** true and "False" otherwise. Each correct answer is worth +1 point; each incorrect answer is worth −½ point; checking "I don't know" is worth +¼ point; and flipping a coin is (on average) worth +¼ point.

☒ Yes ☐ No ☐ IDK — If the moon is made of cheese, then Jeff is the Queen of England.

$p \to q \equiv \neg p \vee q$

☐ Yes ☒ No ☐ IDK — The language $\{0^m 1^n \mid m, n \geq 0\}$ is not regular.

$0^* 1^*$

☐ Yes ☒ No ☐ IDK — For all languages L, the language L^* is regular.

☒ Yes ☐ No ☐ IDK — For all languages $L \subset \Sigma^*$, if L is regular ~~recognized by a DFA~~, then $\Sigma^* \setminus L$ is regular ~~can be represented by a regular expression.~~

☐ Yes ☒ No ☐ IDK — For all languages L and L', if $L \cap L' = \varnothing$ and L' is not regular, then L is regular.

$L = 0^{prime}$ $L' = 0^{prime} 1$

☐ Yes ☒ No ☐ IDK — For all languages L, if L is not regular, then L does not have a finite fooling set.

$F = \varnothing$

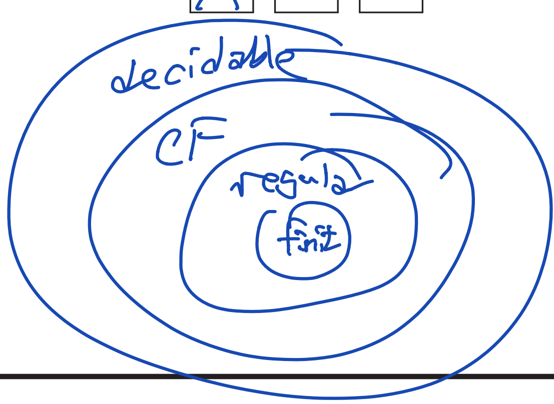
☒ Yes ☐ No ☐ IDK — Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary **DFA**s with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \cap L(M') = \varnothing$.

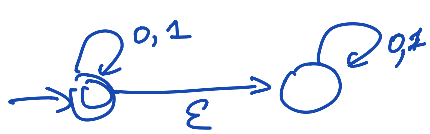
☐ Yes ☒ No ☐ IDK — Let $M = (\Sigma, Q, s, A, \delta)$ and $M' = (\Sigma, Q, s, Q \setminus A, \delta)$ be arbitrary **NFA**s with identical alphabets, states, starting states, and transition functions, but with complementary accepting states. Then $L(M) \cap L(M') = \varnothing$.

☒ Yes ☐ No ☐ IDK — For all context-free languages L and L', the language $L \bullet L'$ is also context-free.

$A \to BC$

☒ Yes ☐ No ☐ IDK — Every non-context-free language is non-regular.





CS/ECE 374 A ✧ Spring 2018 **Fake Midterm 2 Problem 2**	Gradescope name: Flash

For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either ***prove*** that the language is regular or ***prove*** that the language is not regular. ***Exactly one of these two languages is regular.*** Both of these languages contain the string 00110100000110100.

1. $\{0^n w 0^n \mid w \in \Sigma^+ \text{ and } n > 0\}$ $= 0\,\Sigma^+ 0$

Yes: 000101011000

$\underbrace{0}\underbrace{00100000}_{w}\underbrace{0}$

regular!

2. $\{w 0^n w \mid w \in \Sigma^+ \text{ and } n > 0\}$ Not regular!

$\underbrace{11111}_{w}\ 0\ \underbrace{11111}_{w}$

Let $F = 1^+$

pick $x, y \in F$ arbitrarily, $x \neq y$

then $x = 1^i$ $y = 1^j$ for some $i \neq j$

Let $z = 01^i$

$xz = 1^i 0 1^i \in L$ ✓

$yz = 1^j 0 1^i \notin L$ ✓

So F is inf. fooling set for L $\square$

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Fake Midterm 2 Problem 3	

For any string $w \in \{0,1\}^*$, let *collapse*(w) be the string obtained from w by collapsing each run of consecutive 0s to a single 0, and collapsing each run of consecutive 1s to a single 1. For example, *collapse*$(000001110100000) = 01010$.

(a) Give a formal recursive definition of the function *collapse*.

$$collapse(w) = \begin{cases} \varepsilon & w = \varepsilon \\ 0 & w = 0 \\ 1 & w = 1 \\ collapse(0x) & w = 00x \\ 0 \cdot collapse(1x) & w = 01x \\ 1 \cdot collapse(0x) & w = 10x \\ collapse(1x) & w = 11x \end{cases}$$

$\{collapse(w) \mid w \in L\}$

$\neq$

(b) ***Prove*** that for any regular language L, the language $\underbrace{\text{UNCOLLAPSE}(L)}_{L'} := \{w \in \{0,1\}^* \mid collapse(w) \in L\}$ is also regular.

Given DFA $M = (Q, s, A, \delta)$ for L

Build ~~N~~ D FA $M' = (Q', s', A', \delta')$ for L' as follows:

$$Q' = Q \times \underbrace{\{0,1\}}_{\text{last symbol read}} \cup \{s'\}$$

s'

$$A' = A \times \{0,1\} \cup \underbrace{\{s'\}}_{\text{if } s \in A}$$

$$\delta'((q,b),a) = \begin{cases} \cancel{\{}(q,b)\cancel{\}} & \text{if } a = b \\ \cancel{\{}(\delta(q,a),a)\cancel{\}} & \text{else} \end{cases}$$

$$\delta'(s',a) = \cancel{\{}(\delta(s,a),a)\cancel{\}}$$

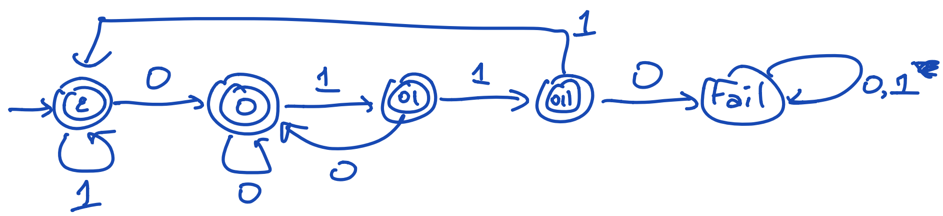
CS/ECE 374 A ✧ Spring 2018 Fake Midterm 2 Problem 4	Gradescope name:

For each of the following languages L, give a regular expression that represents L ***and*** describe a DFA that recognizes L. You do ***not*** need to prove that your answers are correct.

(a) The set of all strings in $\{0, 1\}^*$ that do not contain the substring 0110.

Yes
ε
1
0
1111
000
0010100
1100101111
11110010001110010111011

NO
0110
0110101
10110110
110110110



$$1^*\left(0^+\left(1+111^+\right)\right)^*0^*1^*$$

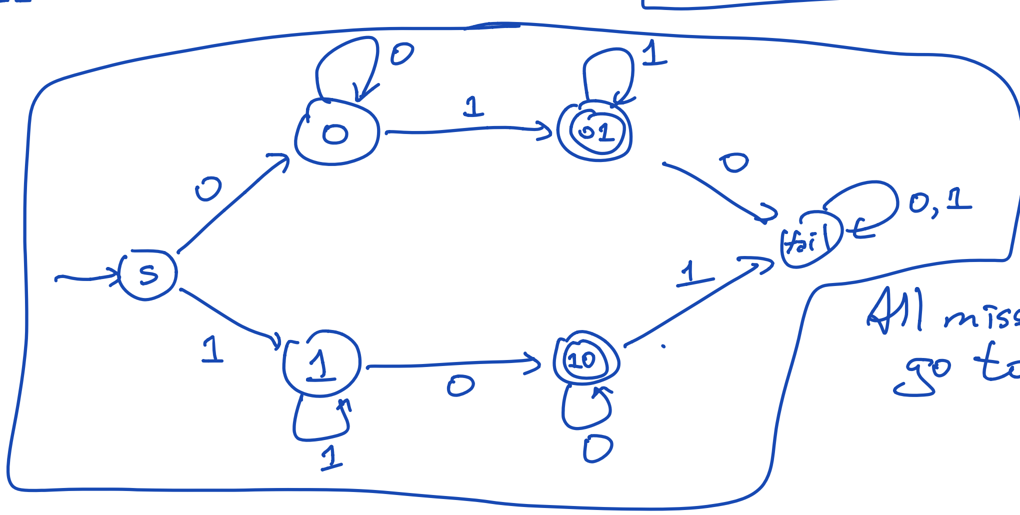
(b) The set of all strings in $\{0, 1\}^*$ that contain exactly one of the substrings 01 or 10.

Yes
0001111
110000
0__01__1
1__10__0

NO
ε
0000
0011100
0_01_10_0

$$0^+1^+ + 1^+0^+$$

$$\boxed{0^*011^* + 1^*100^*}$$



All missing transitions go to fail state

CS/ECE 374 A ✧ Spring 2018 Fake Midterm 2 Problem 5	Gradescope name:

Consider the language L of all strings $\{0,1\}^*$ in which the number of 0s is even, the number of 1s is divisible by 3, and the total number of characters is divisible by 5. For example, the strings 01011 and 0000000000 are in L, but the strings 01001 and 10101010 are not.

Formally describe a DFA $M = (Q, \{0,1\}, a, S, \delta)$ that recognizes L. ***Do not attempt to draw the DFA. Do not use the phrase "product construction".*** Instead, formally and explicitly specify each of the the components Q, s, A, and δ.

This is a ~~product~~ ~~construction~~ of three DFAs:

- #0s mod 2 = 0
- #1s mod 3 = 0
- length mod 5 = 0

so we need three variables

$$Q = \{0,1\} \times \{0,1,2\} \times \{0,1,2,3,4\}$$

$$s = (0,0,0)$$

$$A = \{(0,0,0)\}$$

$$\delta((x,y,z),a) = \begin{cases} ((x+1) \bmod 2,\ y,\ (z+1) \bmod 5) & \text{if } a = 0 \\ (x,\ (y+1) \bmod 3,\ (z+1) \bmod 5) & \text{if } a = 1 \end{cases}$$