
Submission instructions as in previous homeworks.

10 (100 PTS.) **OLD Homework problem (not for submission):**

Aberrant.

- 10.A.** (25 PTS.) Prove that the following language is not regular by providing a fooling set. You need to prove an infinite fooling set and also prove that it is a valid fooling set. For $\Sigma = \{a, b\}$, the language is

$$L = \{ww \mid w \in \Sigma^+\}.$$

- 10.B.** (25 PTS.) Same as (A) for the following language. Recall that a **run** in a string is a maximal non-empty substring of identical symbols. Let L be the set of all strings in Σ^* that contains two distinct runs of equal length. A few examples about L :

- L contains any string of the form $b^i a^+ b^+ a^i$.
- L contains any string of the form $b^i a^+ b^i$.
- L does not contain the strings $abbaaa$, $abbaaabbbb$.

- 10.C.** (25 PTS.) Suppose you are given two languages L, L' that are not regular but such that $L' \setminus L$ is regular. Prove that $L \cup L'$ is not regular. (Hint: Use closure properties of regular languages.)

- 10.D.** (15 PTS.) Provide a counter-example for the following claim:

Claim: Consider two languages L and L' . If \bar{L} is not regular, L' is regular, and $L \cup L'$ is regular, then $L \cap L'$ is regular.

- 10.E.** (10 PTS.) (Slightly harder¹) Same as (A) for $L = \{0^{n^4} \mid n \geq 3\}$.

11 (100 PTS.) **OLD Homework problem (not for submission):**

Grammar it.

Describe a context free grammar for the following languages. Clearly explain how they work and the role of each non-terminal. Unclear grammars will receive little to no credit.

- 11.A.** (40 PTS.) $\{a^i b^j c^k d^\ell e^t \mid i, j, k, \ell, t \geq 0 \text{ and } i + j + k + t = \ell\}$.

- 11.B.** (60 PTS.) (Harder.) $L = \{z \in \{a, b, c\}^* \mid \text{there is a suffix } y \text{ of } z \text{ s.t. } \#_a(y) > \#_b(y)\}$.
(Hint: First solve for the case that z has no cs .)

12 (100 PTS.) **OLD Homework problem (not for submission):**

As easy as 1,2,3,6.

$$\text{Let } L = \{a^i b^j c^k \mid k = i + j\}.$$

¹Feel free to use IDK.

- 12.A.** (20 PTS.) Prove that L is context free by describing a grammar for L .
- 12.B.** (80 PTS.) Prove that your grammar is correct. (See extra problems for an example of how this is done.)