I February 5, 2019 11:00 AM II

Last Time:

Proving Non-regularity

- fooling sets

Other ways to prove non-regular

- combine with closure properties

Ex $L = \{ w \in \{0,1,2\}^* \mid \#_0(w) \neq \#_1(w) \}$ is not regular.

Pf: Suppose L is regular.

0^*1^* is regular.

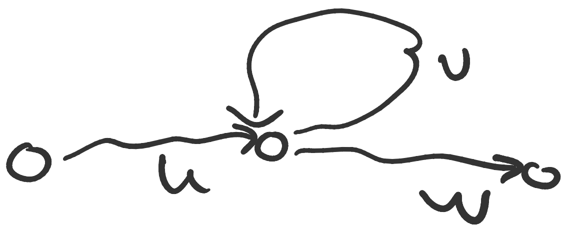
$\bar{L} \cap 0^*1^*$ is also regular

$= \{ 0^n 1^n \mid n \geq 0 \}$

known to not regular.

Contra! $\square$

- Pumping lemma (skipped)



Rmk - appl of Myhill-Nerode to test if 2 DFAs are equiv.
& 2 reg exprs are equiv.

Context-Free Langs (CFL)

(langs generated by recursive replacement rules)

Ex (i) $\{0^n 1^n \mid n \geqslant 0\}$

Rules: $S \to 0S1 \mid \varepsilon$

e.g. to generate 000111,

$S \rightsquigarrow 0S1 \rightsquigarrow 00S11$

$\rightsquigarrow 000S111$

$\rightsquigarrow 000111$

(ii) $\{$ all palindromes in $\{0,1\}^* \}$

$(\{w \mid w = w^R\})$

Rules: $S \to 0S0 \mid 1S1 \mid \varepsilon \mid 0 \mid 1$

e.g. 01110

$S \rightsquigarrow 0S0 \rightsquigarrow 01S10 \rightsquigarrow 01110$

Formal Def A context-free grammar (CFG) is a specified as $G = (V, T, P, S)$

where V is finite set of variables ("non-terminals")

T is finite alphabet ("terminals")

P is finite set of rules ("productions")

P is finite set of rules ("productions"), each of the form $A \to \alpha$ where $A \in V$, $\alpha \in (V \cup T)^*$

$S \in V$ is the <u>start</u> symbol

<u>Ex</u> (i) $V = \{S\}$, $T = \{0,1\}$
$P = \{ S \to 0S1,\ S \to \varepsilon \}$

<u>Def</u> $\alpha_1 \underset{G}{\rightsquigarrow} \alpha_2$ (α_1 derives α_2 in one step)

iff $\alpha_1 = \beta A \delta$, $\alpha_2 = \beta \gamma \delta$,
and $A \to \gamma$ is in P
for some $\beta, \delta \in (V \cup T)^*$.

<u>Ex1</u> $0S1 \rightsquigarrow 00S11$

<u>Def</u> $\alpha_1 \overset{k}{\underset{G}{\rightsquigarrow}} \alpha_2$ (α_1 <u>derives</u> α_2 in k steps)

iff $\begin{cases} \alpha_1 = \alpha_2 & \text{if } k = 0 \\ \alpha_1 \underset{G}{\rightsquigarrow} \beta,\ \beta \overset{k-1}{\underset{G}{\rightsquigarrow}} \alpha_2 & \text{if } k > 0 \text{ for some } \beta \end{cases}$

$\alpha_1 \overset{*}{\underset{G}{\rightsquigarrow}} \alpha_2$ iff $\alpha_1 \overset{k}{\underset{G}{\rightsquigarrow}} \alpha_2$ for some k

$$L(G) = \{ x \in T^* \mid S \overset{*}{\underset{G}{\rightsquigarrow}} x \}$$

generated

lang generated by G

L is a CFL if $L = L(G)$ for some CFG G.

Ex a) $(1+01)^*(1+10)^* + 1^*0$

A for $(1+01)^*$

B for $(1+10)^*$

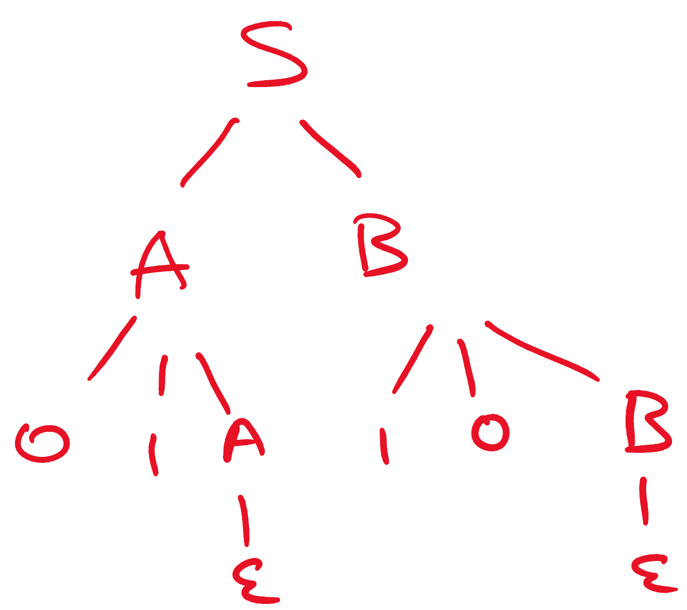
C for 1^*0

$$S \to AB \mid C$$
$$A \to 1A \mid 01A \mid \varepsilon$$
$$B \to 1B \mid 10B \mid \varepsilon$$
$$C \to 1C \mid 0$$

more generally, any regular lang is CFL.

e.g. $0110 \in L(G)$

$$S \rightsquigarrow AB \rightsquigarrow 01AB \rightsquigarrow 01B \rightsquigarrow 0110B \rightsquigarrow 0110$$



"derivation tree" / "parse tree"

b) $\{ 0^i 1^j 2^k \mid j > i+k \}$

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idea: $\underbrace{0^i 1^i}_{A} \; \underbrace{1^{j-(i+k)}}_{B} \; \underbrace{1^k 2^k}_{C}$

$S \to ABC$

$A \to 0A1 \mid \varepsilon$

$C \to 1C2 \mid \varepsilon$

$B \to 1B \mid 1$

c) $\{ 0^i 1^i 2^i \mid i \geq 0 \}$

impossible

d) $\{ ww \mid w \in \{0,1\}^* \}$ e.g. 011011

impossible

e) $\{ x \in \{0,1\}^* \mid x$ is <u>not</u> palindrome$\}$

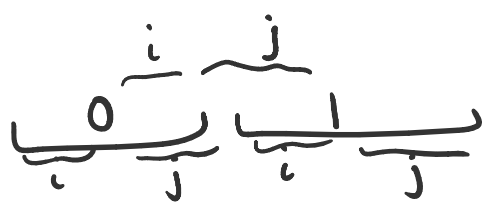
e.g. 01011110

$S \to 0S0 \mid 1S1 \mid 0A1 \mid 1A0$

$A \to 0A \mid 1A \mid \varepsilon$

f) complement of $\{ ww \mid w \in \{0,1\}^* \}$

possible!



g) all strings of balanced parentheses

e.g. (( ) ( )) ( )

e.g. $(\,()\,()\,)\,()$

$$S \to (S) \mid SS \mid \varepsilon$$

h) all strings w. equal # 0's, 1's

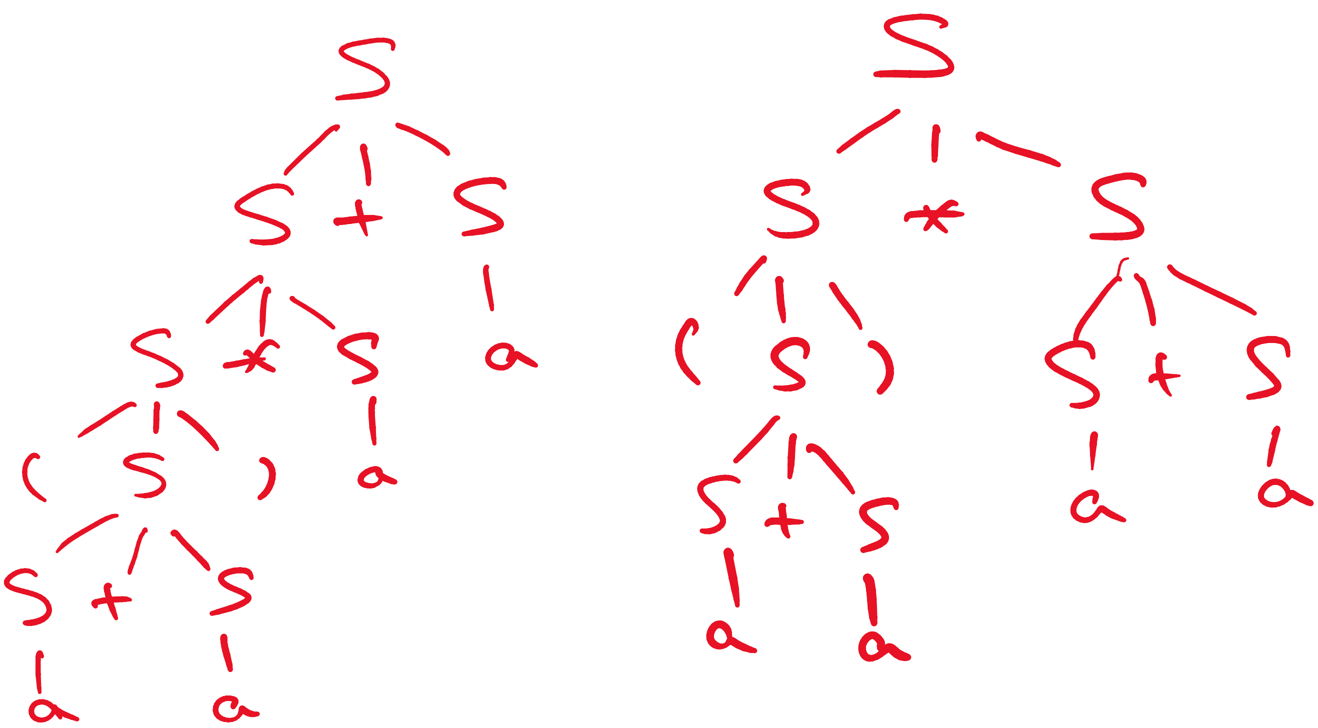
$$S \to 0S1 \mid 1S0 \mid SS \mid \varepsilon$$

(requires induction pf – see slides for exs)

i) all arithmetic exprs
e.g. $(a+a)*a+a$

$$S \to S+S \mid S*S \mid (S) \mid a$$

but ambiguous
(∃ string w. 2 diff. deriv trees)



alternative sol'n:

$$S \to S + T \mid \cancel{S}\; T$$

$$T \to T * F \mid F$$

$$F \to (S) \mid a \qquad \text{(unambiguous)}$$

Rmk: CFL $\Longleftrightarrow$ langs accepted by nondet pushdown automata (PDA)