February 7, 2019 10:59 AM

Last Time:

CFL $\longleftrightarrow$ nondet. pushdown automata

stack

$0^n 1^n$

palindromes

$\{ww \mid w \in \{0,1\}^*\}$

Closure Props

Thm If L_1, L_2 are CFL, then so are $L_1 \cup L_2$, $L_1 L_2$, L_1^*

$S \to S_1 \mid S_2$ $\qquad$ $S \to S_1 S_2$

$S \to S S_1 \mid \varepsilon$

Thm If L_1 is CFL, L_2 is regular, then $L_1 \cap L_2$ is CFL.

product construct. with PDA & DFA

Rmk: not closed under intersection

$L_1 = \{0^i 1^i 2^k \mid i, k \geq 0\}$ CFL

$L_2 = \{0^i 1^k 2^k \mid i, k \geq 0\}$ CFL

$L_1 \cap L_2 = \{0^i 1^i 2^i \mid i \geq 0\}$ not CFL not closed under complement

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Turing Machines (TM)

how to define all that can be computed?

Turing (1936) - one of simplest models resembling actual computer

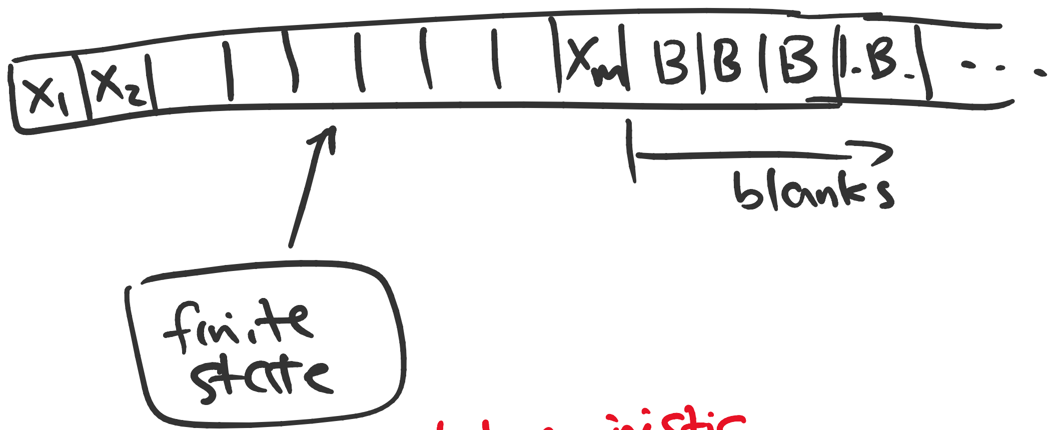
main features

- unlimited memory
- can both read & write
- can go back & forth in memory

one specific way

- unbounded tape
- read/write one symbol at current position ("head")
- move head one position left/right

tape



Formal Def

A TM is specified as

deterministic

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, q_{acc}, q_{rej})$$

where

- Q is a finite set of states
- Σ is finite input alphabet
- Γ is finite tape alphabet $(\Sigma \subseteq \Gamma)$
- $q_0 \in Q$ is start state
- $B \in \Gamma \setminus \Sigma$ is blank symbol

$B \in \Gamma \setminus \Sigma$ is <u>blank symbol</u>

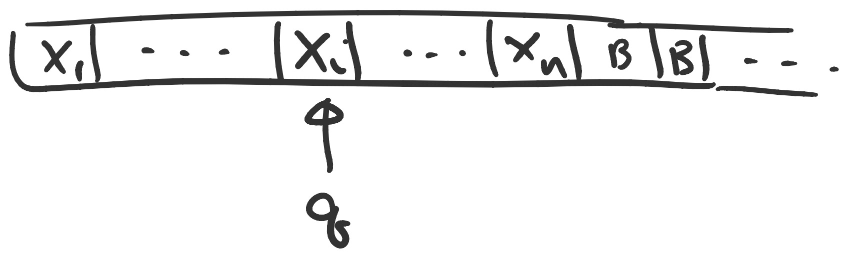
$q_{acc} \in Q$ is (unique) accept state

$q_{rej} \in Q$ " " reject "

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R, S\}$$

current state | symbol read | next state | symbol to write | move left/right/stay

Def A <u>configuration</u> (also called an <u>instantaneous description</u> (ID)) is a string $X_1 \cdots X_{i-1}\, q\, X_i \cdots X_n$ $(q \in Q,\ X_1, \dots, X_n \in \Gamma)$



Def (Single move) Let $q \neq q_{acc}, q_{rej}$.

If $\delta(q,a) = (q', a', S)$,

$$\alpha q a \beta \underset{M}{\rightsquigarrow} \alpha q' a' \beta \qquad \text{for any } \alpha, \beta \in \Gamma^*$$

If $\delta(q,a) = (q', a', R)$,

$$\alpha q a \beta \underset{M}{\rightsquigarrow} \alpha a' q' \beta$$

(special case: $\alpha q a \underset{M}{\rightsquigarrow} \alpha a' q' B$)

If $\delta(q,a) = (q', a', L)$,

$$\alpha b q a \beta \underset{M}{\rightsquigarrow} \alpha q' b a' \beta$$

(special case: $q a \beta \underset{M}{\rightsquigarrow}$ crash) fall off left end

Def (Multiple moves)

$C \overset{*}{\underset{M}{\rightsquigarrow}} C'$ iff $C \underset{M}{\rightsquigarrow} C_1 \underset{M}{\rightsquigarrow} C_2 \cdots \underset{M}{\rightsquigarrow} C_{k-1} \underset{M}{\rightsquigarrow} C'$

for some $C_1, \ldots, C_{k-1}$

Def On input $x \in \Sigma^*$,

M <u>accepts</u> x iff $q_0 x \overset{*}{\underset{M}{\rightsquigarrow}} \alpha\, q_{acc}\, \beta$

for some $\alpha, \beta \in \Gamma^*$

M <u>rejects</u> x iff $q_0 x \overset{*}{\underset{M}{\rightsquigarrow}} \alpha\, q_{rej}\, \beta$

for some α, β

M <u>crashes</u> iff $q_0 x \overset{*}{\underset{M}{\rightsquigarrow}}$ crash

M does <u>not halt</u> iff $q_0 x \underset{M}{\rightsquigarrow} C_1 \underset{M}{\rightsquigarrow} C_2 \underset{M}{\rightsquigarrow} C_3 \rightsquigarrow \cdots$

for infinite seq.

Def $L(M) = \{ x \in \Sigma^* \mid M \text{ accepts } x \}$

(lang accepted by M)

L is <u>recursively enumerable</u> (r.e.)

iff $L = L(M)$ for some TM M.

L is <u>recursive</u> (also called <u>decidable</u>)

iff $L = L(M)$ for some TM M

that <u>halts</u> on all input.

Rmk: diagram notation

means $\delta(q,a) = (q', a', D)$

$D \in \{L, R, S\}$

no arc means $\delta(q,a) = (q_{rej}, a, S)$

Ex1

$\{ 0^n 1^n 2^n \mid n \geq 1 \}$

$\Gamma = \{0, 1, 2, X, Y, Z, \text{blank}\}$

X X X X Y Y Y Y Z Z Z Z

$\cancel{0}$ $\cancel{0}$ $\cancel{0}$ $\cancel{0}$ $\cancel{1}$ $\cancel{1}$ $\cancel{1}$ $\cancel{1}$ $\cancel{2}$ $\cancel{2}$ $\cancel{2}$ $\cancel{2}$ BB...

trick: marking

idea: repeat:

1. change first 0 to X, go R to find first 1
2. change 1 to Y, go R to find first 2
3. change 2 to Z, go L back to rightmost X

until no more 0's

check no more 1's, 2's