2. Regular expressions

For each of the following languages over the alphabet $\{0, 1\}$, give an equivalent regular expression.

15. ⟨⟨*S14*⟩⟩ The set of all strings in 0^*1^* whose length is divisible by 3.

$0^i 1^j$

$i + j = 0 \bmod 3 \to i = 0 \bmod 3 \;;\; j = 0 \bmod 3$

$\to i = 1 \bmod 3 \;;\; j = 2 \bmod 3$

$\to i = 2 \bmod 3 \;;\; j = 1 \bmod 3$

$$(000)^*(111)^* + 0(000)^*11(111)^*$$

$$+ 00(000)^*1(111)^*$$

3. Direct DFA construction.

Draw or formally describe a DFA that recognizes each of the following languages. If you draw the DFA you may omit transitions to a reject/junk state.

26. ⟨⟨*S14*⟩⟩ The set of all strings in 0^*1^* whose length is divisible by 3.

DFA 1: accepts $0^* 1^*$ → Intersect.

DFA 2: accept $\omega \mid |\omega| = 0 \bmod 3$

4. Fooling sets

Prove that each of the following languages is *not* regular.

42. $\{w\#x\#y \mid w, x, y \in \Sigma^* \text{ and } w, x, y \text{ are not all equal}\}$

Fooling set: $\{0^i \# 0^i \# \mid i \geq 0\}$

(1) Infinite

(2) $0^i \# 0^i \# 0^i \notin L$

$0^j \# 0^j \# 0^i \in L$

5. Regular or Not?

For each of the following languages, either prove that the language is regular (by describing a DFA, NFA, or regular expression), or prove that the language is not regular (using a fooling set argument). Unless otherwise specified, all alphabets are 0, 1.

44. ⟨⟨*F14*⟩⟩ The set of all strings in $\{0,1\}^*$ in which the substrings 00 and 11 appear the same number of times. (For example, the substrings 00 and 11 each appear three times in the string 1100001101101.)

Not Regular

Fooling set: $\{0^i \mid i \geq 1\}$

(1) Infinite

(2) $0^i 1^i \in L \Rightarrow \#_{00}(0^i 1^i) = \#_{11}(0^i 1^i) = i-1$

$0^j 1^i \notin L \Rightarrow \#_{00}(0^j 1^i) = j-1$

$\neq \#_{11}(0^j 1^i) = i-1$

5. Regular or Not?

For each of the following languages, either prove that the language is regular (by describing a DFA, NFA, or regular expression), or prove that the language is not regular (using a fooling set argument). Unless otherwise specified, all alphabets are 0, 1.

43. *⟨⟨F14⟩⟩* The set of all strings in $\{0,1\}^*$ in which the substrings 01 and 10 appear the same number of times. (For example, the substrings 01 and 01 each appear three times in the string 1100001101101.)

Regular: count of 01 & 10 substrings not independent

0111111 ⟶ cannot see another 01 unless we see a 10

$\Delta = |\#_{01}(x) - \#_{10}(x)| \leq 1$

+1
-1
0

0

1

DFA with 6 states: remember Δ & last symbol!

+1 start state.

5. Regular or Not?

For each of the following languages, either prove that the language is regular (by describing a DFA, NFA, or regular expression), or prove that the language is not regular (using a fooling set argument). Unless otherwise specified, all alphabets are 0, 1.

46. ⟨⟨*F14*⟩⟩ $\{wxw \mid w, x \in \Sigma^*\}$ $= L = \Sigma^*$

$L \subseteq \Sigma^*$ ✓

$\Sigma^* \subseteq L \Rightarrow$ let $y \in \Sigma^*$, then $y = wxw$ where
$w = \varepsilon$
$x = y$

$\Rightarrow y \in L$

$\{wxw \mid x \in \Sigma^*, w \in \Sigma^+\}$ NOT REGULAR

Fooling set $= \{1^i 0^i 1 \mid i \geq 1\}$ $1^i 0^i 1\, 1^i 0^i \in L$
$1^j 0^j 1\, 1^i 0^i \notin L$

5. Regular or Not?

For each of the following languages, either prove that the language is regular (by describing a DFA, NFA, or regular expression), or prove that the language is not regular (using a fooling set argument). Unless otherwise specified, all alphabets are 0, 1.

57. $\{w\#x \mid w, x \in \{0,1\}^* \text{ and } w \text{ is a proper substring of } x\}$

$\{0^i \mid i \geq 1\}$ is fooling set

Infinite

$0^i \# 0^i \notin L$

$0^j \# 0^i \in L$

$j > i$

11. True or False (sanity check)

137. ⟨⟨*S14*⟩⟩ For all languages $L \subseteq \Sigma^*$, if L contains all but a finite number of strings of Σ^*, then L is regular.

Let L' be the strings in Σ^* not in L

$$L' = \Sigma^* \setminus L$$

regular finite

regular

regular

11. True or False (sanity check)

155. If $L \subseteq L'$ and L' is not regular, then L is not regular. ⟨⟨*F14*⟩⟩

$L' = \{0^n 1^n\}$

$L = \{01\}$

$L \subseteq L'$

regular

finite

not regular

11. True or False (sanity check)

171. For all languages L, if L is not regular, then L has no finite fooling set. ⟨⟨*F14*⟩⟩

Let L be non regular.

Let F be infinite fooling set

any $F' \subseteq F$ is a fooling set

$\Rightarrow$ F' can be finite

11. True or False (sanity check)

174. $\{0^i 1^j 2^k \mid i + j + k = 374\}$ is regular.

Finite $\Rightarrow$ regular

11. True or False (sanity check)

172. $\{0^i 1^j 2^k \mid i + j - k = 374\}$ is regular. ⟨⟨*S14*⟩⟩

not regular.

Fooling set : $\{0^i 1^i \mid i > 374\}$

$0^i 1^i 2^{2i-374} \in L$ $\quad i + i - (2i - 374) = 374$

$0^i 1^j 2^{2i-374} \notin L$ $\quad 2j - 2i + 374 \neq 374$

$i \neq j$;

11. True or False (sanity check)

175. $\{0^i 1^j 2^k \mid i + j + k > 374\}$ is regular.

Regular: count up to 374

$= \{0^i 0^* 1^j 1^* 2^k 2^* \mid i+j+k = 375\}$.

finite.

OR $L' = \{0^i 1^j 2^k \mid i+j+k \leq 374\} \Rightarrow$ finite $\Rightarrow$ regular

$L = 0^* 1^* 2^* \setminus L'$

reg reg reg

9. Context-Free Grammars

Construct context-free grammars for each of the following languages, and give a brief explanation of how your grammar works, including the language of each non-terminal.

81. All strings in $\{0, 1\}^*$ whose length is divisible by 5.

$S \rightarrow \varepsilon \mid 0S_1 \mid 1S_1$ 0 mod 5.

$S_1 \rightarrow 0S_2 \mid 1S_2$ 1 mod 5

$S_2 \rightarrow 0S_3 \mid 1S_3$ 2 mod 5

$S_3 \rightarrow 0S_4 \mid 1S_4$ 3 mod 5

$S_4 \rightarrow 0S \mid 1S$ 4 mod 5

9. Context-Free Grammars

Construct context-free grammars for each of the following languages, and give a brief explanation of how your grammar works, including the language of each non-terminal.

87. $\{0^{i+j}\#0^j\#0^i \mid i, j \geq 0\}$

$$0^i\underbrace{(0^j\#0^j)}_{A}\#0^i$$

$$A \rightarrow 0A0 \mid \#$$

$$S \rightarrow 0S0 \mid A\#$$

9. Context-Free Grammars

Construct context-free grammars for each of the following languages, and give a brief explanation of how your grammar works, including the language of each non-terminal.

89. $\{w\#0^{\#(0,w)} \mid w \in \{0,1\}^*\}$

$S \rightarrow 0S0 \mid 1S \mid \#$

8. Regular Language Transformations

Let L be an arbitrary regular language over the alphabet $\Sigma = \{0, 1\}$. Prove that each of the following languages over $\{0, 1\}$ is regular. "Describe" does not necessarily mean "draw".

73. $\textsc{OnlyOnes}^{-1}(L) := \{w \mid 1^{\#(1,w)} \in L\}$

DFA for L. $M = (Q, \Sigma, \delta, s, A)$

NFA for $\text{OnlyOnes}^{-1}(L)$: $N = (Q, \Sigma, \delta', s, A)$

$$\delta'(q, 1) = \delta(q, 1)$$

$$\delta'(q, 0) = \{q \cup \underbrace{\delta(q, 0)}_{\text{not needed}}\}$$

Idea $\to$ add as many zeros between ones.