CS/ECE 374 A (Spring 2020)
Homework 5 (due Mar 5 Thursday at 10am)

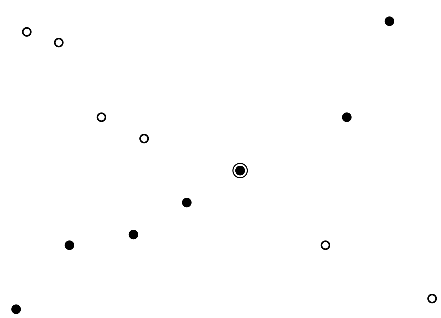
Instructions: As in previous homeworks.

Problem 5.1: A *point* p in 2D is specified by its x-coordinate $p.x$ and y-coordinate $p.y$.

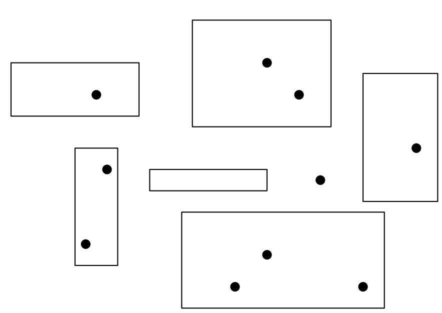
An array of points $P[1,\ldots,n]$ in 2D is *ascending* iff $P[1].x < P[2].x < \cdots < P[n].x$ and $P[1].y < P[2].y < \cdots < P[n].y$. An array of points $Q[1,\ldots,m]$ is *descending* iff $Q[1].x < Q[2].x < \cdots < Q[m].x$ and $Q[1].y > Q[2].y > \cdots > Q[m].y$.

Describe[1] an efficient algorithm for the following problem: given an ascending array $P[1,\ldots,n]$ and a descending array $Q[1,\ldots,m]$, find a common point, i.e., some i,j with $P[i] = Q[j]$, if one exists. [Note that if one exists, the answer must be unique (why?).] Aim for $O(\log n + \log m)$ running time. [A slower algorithm with $O(\log n \log m)$ running time will still get partial credit.]

(For example: for the ascending sequence $\langle (1,0),(4,1),(5,4),(8,5),(13,7)\rangle$ and the descending sequence $\langle (0,11),(2,10),(8,5),(9,2),(11,1)\rangle$, there is a common point $(8,5)$.)



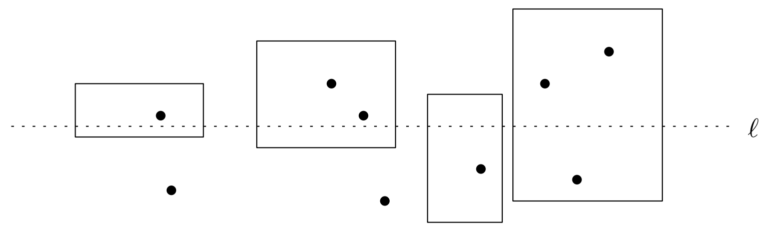
Problem 5.2: Consider the following problem: We are given a set R of n_1 rectangles in 2D, where the sides of the rectangles are parallel to the x- and y-axes (i.e., the rectangles are not rotated) and the rectangles do not overlap. We are also given a set P of n_2 points in 2D. For every point $p \in P$, we want to find the rectangle $r_p \in R$ that contains p. (Note that if p is not covered by any of the rectangles in R, then r_p is undefined, but otherwise r_p is unique because of the nonoverlapping assumption.) Let $n = n_1 + n_2$.



[1] As usual, this means giving a pseudocode description (not actual code!), along with explanation or justification of correctness (especially if it is not obvious), and analysis of the running time.

(a) (3.0 points) First give an $O(n \log n)$-time algorithm for the special case when all rectangles of R are assumed to intersect a given horizontal line ℓ.

[Hint: sort...]



(b) (7.0 points) Now describe an algorithm to solve the general problem in $O(n \log^2 n)$ time or better.

[Hint: use divide-and-conquer and part (a) as a subroutine.][2]

Problem 5.3: Consider the following problem: given a number n, compute $n!$ (the factorial). Here, we measure running time in terms of the number of bit operations. Notice that $n!$ has $\Theta(\log(n!)) = \Theta(n \log n)$ bits.

(a) (2.0 points) Recall that Karatsuba's algorithm can multiply two k-bit integers in $O(k^{1.59})$ time. Using Karatsuba's algorithm as a subroutine, show that we can multiply a k-bit integer with an ℓ-bit integer ($\ell \geq k$) in $O(\ell k^{0.59})$ time.

(b) (2.0 points) Analyze the naive iterative algorithm to compute $n!$ (i.e., for $i = 1$ to n, multiply the current answer with i). Show that using (a), this algorithm runs in $O(n^2 \log^{1.59} n)$ time.

(c) (6.0 points) Next, design and analyze a faster divide-and-conquer algorithm to compute $n!$, running in $O(n^{1.59} \log^{1.59} n)$ time.

[Hint: solve a more general problem, of computing $(M + 1)(M + 2) \cdots (M + n)$ for any given n and M...]

[2] As usual, in a multi-part question, if you are unable to solve (a), you can still do (b) under the assumption that (a) has been solved.