HW 1

Due on **Monday, January 27, 2020 at 11pm**

CS/ECE 374 B: Algorithms & Models of Computation, Spring 2020 Version: **1.05**

Submission instructions as in previous homeworks.

1 (100 PTS.) Review

1.A. (50 PTS.) Suppose S is a set of 111 integers. Prove that there is a subset $S' \subseteq S$ of at least 11 numbers such that the difference of any two numbers in S' is a multiple of 11.

1.B. (50 PTS.) The famous Basque computational arborist Gorka Oihanean has a favorite 26-node binary tree, in which each node is labeled with a letter of the alphabet. Inorder and postorder traversals of his tree visits the nodes in the following orders:

Inorder: $U\ C\ N\ O\ B\ I\ E\ L\ Z\ T\ F\ D\ S\ H\ W\ V\ Q\ A\ X\ G\ R\ J\ P\ K\ M\ Y$

Postorder: $U\ C\ O\ I\ E\ B\ L\ F\ S\ D\ H\ T\ Z\ N\ V\ A\ R\ G\ J\ X\ M\ K\ P\ Y\ Q\ W$

List the nodes in Professor Oihanean's tree according to a preorder traversal.

2 (100 PTS.) A recurrence.

Consider the recurrence

$$T(n) = \begin{cases} T(\lfloor n/3 \rfloor) + 4T(\lfloor n/6 \rfloor) + n & n \geq 6 \\ 1 & n < 6. \end{cases}$$

Prove by induction that $T(n) = O(n \log n)$. (Recall that you need to show that $T(n) \leq c_1 n \log n + c_2$ for $n \geq 1$ where $c_1, c_2 \geq 0$ are some fixed but suitably chosen constants.

3 (100 PTS.) Languages

Let $L \subseteq \{0, 1\}^*$ be a language defined recursively as follows:

(i) The string 0 is in L.
(ii) For any string x in L, the string $x1$ is also in L.
(iii) For any string x in L, the string $1x$ is also in L.
(iv) For any strings x and y in L, the string $x0y$ is also in L.
(v) These are the only strings in L.

Let $\#_0(w)$ denote the number of times 0 appears in string w and $\#_1(w)$ denote the number of times 1 appears in string w. You may assume without proof that $\#_0(xy) = \#_0(x) + \#_0(y)$, for any strings x, y.

3.A. (50 PTS.) Prove by induction that every string $w \in L$ contains an odd number of 0s.

3.B. (50 PTS.) Let $L' \subseteq \{0, 1\}^*$ be the language of strings with an odd number of 0s. Prove that $L = L'$.