1 Let $M = (\Sigma, Q, s, A, \delta)$ be an arbitrary DFA that accepts $L(M)$. Construct and NFA N that accepts the language $\mathit{half}(L(M)) := \{w \mid ww \in L(M)\}$.

Solution:

We define a new NFA $N = (\Sigma, Q', s', A', \delta')$ with ε-transitions that accepts $\mathit{half}(L)$, as follows:

$$Q' = (Q \times Q \times Q) \cup \{s'\}$$
$$s' \text{ is an explicit state in } Q'$$
$$A' = \{(h, h, q) \mid h \in Q \text{ and } q \in A\}$$

$$\delta'(s', \varepsilon) = \{(s, h, h) \mid h \in Q\}$$
$$\delta'((p, h, q), a) = \{(\delta(p, a), h, \delta(q, a))\}$$

N reads its input string w and simulates M reading the input string ww. Specifically, N simultaneously simulates two copies of M, one reading the left half of ww starting at the usual start state s, and the other reading the right half of ww starting at some intermediate state h.

- The new start state s' non-deterministically guesses the "halfway" state $h = \delta^*(s, w)$ without reading any input; this is the only non-determinism in N.
- State (p, h, q) means the following:
  - The left copy of M (which started at state s) is now in state p.
  - The initial guess for the halfway state is h.
  - The right copy of M (which started at state h) is now in state q.
- N accepts if and only if the left copy of M ends at state h (so the initial non-deterministic guess $h = \delta^*(s, w)$ was correct) and the right copy of M ends in an accepting state.

Rubric: 5 points =

+ 1 for a formal, complete, and unambiguous description of a DFA or NFA
  - No points for the rest of the problem if this is missing.
+ 3 for a correct NFA
  - -1 for a single mistake in the description (for example a typo)
+ 1 for a *brief* English justification. We explicitly do *not* want a formal proof of correctness, but we do want one or two sentences explaining how the NFA works.