1 Let $L = \{w \in \{a, b\}^* \mid a \text{ appears in some position } i \text{ of } w \text{, and a } b \text{ appears in position } i+2\}$.

1.A. Create an NFA N for L with at most four states.

1.B. Using the "power-set" construction, create a DFA M from N. Rather than writing down the sixteen states and trying to fill in the transitions, build the states as needed using "incremental subset" construction, because you won't end up with unreachable or otherwise superfluous states.

1.C. Now directly design a DFA M' for L with only five states, and explain the relationship between M and M'.

2 Use Thompson's algorithm to create an NFA for the following regular expressions:

2.A. $(0+1)^*$

2.B. $01^* + 10^*$

3 Consider the following recursively defined function on strings:

$$stutter(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ aa \bullet stutter(x) & \text{if } w = ax \text{ for some symbol } a \text{ and some string } x \end{cases}$$

Intuitively, $stutter(w)$ doubles every symbol in w. For example:

- $stutter(PRESTO) = PPRREESSTTOO$
- $stutter(HOCUS\diamond POCUS) = HHOOCCUUSS\diamond\diamond PPOOCCUUSS$

Let L be an arbitrary regular language.

1. Prove that the language $stutter^{-1}(L) := \{w \mid stutter(w) \in L\}$ is regular.
2. Prove that the language $stutter(L) := \{stutter(w) \mid w \in L\}$ is regular.

Work on these later:

4 Consider the following recursively defined function on strings:

$$evens(w) := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ \varepsilon & \text{if } w = a \text{ for some symbol } a \\ b \cdot evens(x) & \text{if } w = abx \text{ for some symbols } a \text{ and } b \text{ and some string } x \end{cases}$$

Intuitively, $evens(w)$ skips over every other symbol in w. For example:

- $evens(EXPELLIARMUS) = XELAMS$
- $evens(AVADA\diamond KEDAVRA) = VD\diamond EAR$.

Once again, let L be an arbitrary regular language.

1. Prove that the language $evens^{-1}(L) := \{w \mid evens(w) \in L\}$ is regular.
2. Prove that the language $evens(L) := \{evens(w) \mid w \in L\}$ is regular.