1 Given a directed graph $G = (V, E)$ with non-negative edge lengths $\ell(e), e \in E$ and a node $s \in V$, describe an algorithm to find the length of a shortest cycle containing the node s.

2 Suppose we have a collection of cities and different airlines offer flights between various pairs of cities. Some airlines only fly between some pairs of cities. Some pairs of cities are served by many airlines. Each airline charges perhaps different amounts for their one-way tickets.

- Suppose you'd like to get from City A to City B at the least total cost. Describe an efficient solution. (Your solution may change planes to a different airline as needed.)
- It turns out that airports charge usage taxes. Different airports may charge different amounts in tax. Your cost of traveling from A to B now includes all of the flight costs, plus all of the taxes of the airports that you stopover along the way from A to B. Model this as a graph problem and give an efficient solution to find the least cost way to get from A to B.

3 Describe and analyze an algorithm to compute the shortest path from vertex s to vertex t in a directed graph with weighted edges, where exactly *one* edge $u \to v$ has negative weight. First check whether G has a negative length cycle. Then, find the shortest path length from s to t. (**Hint:** Modify the input graph and run Dijkstra's algorithm.)