Backtracking

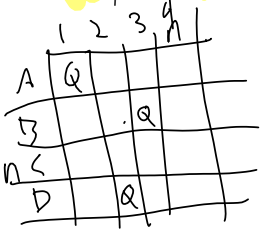
Tuesday, March 3, 2020 3:33 PM

Kinds of problems:

- Decision problem:
  Yes/no
  ex. Given a graph G, and source s, target t, does there exist a path $s \rightsquigarrow t$
- Search problem:
  Find an acceptable solution from a solution space.
  Find a path from $s \rightsquigarrow t$
  $G = (V, E)$
  Solution space = $P(E)$
- Optimization problem:
  e.g. Find shortest path $s \rightsquigarrow t$

  Given NFA M does M accept string s?
- approximation

8-queens n-queens



Place n queens on board such none can attack each other

$\Sigma = \{\epsilon, Q\}$

Conf: $\Sigma^{n \times n}$

Good: Conf $\rightarrow \{0,1\}$

1. diag. n
2. cols. n
3. rows n

each ex. 1

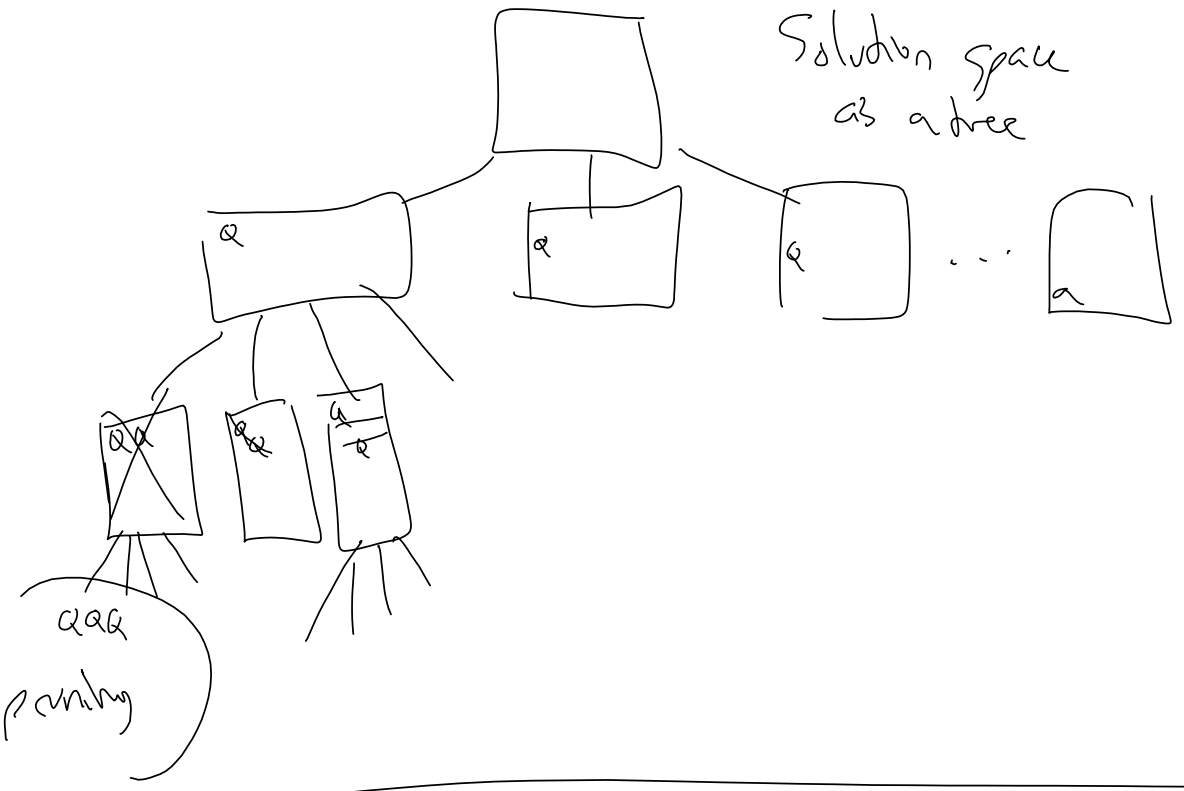
$T_{Good}(n) = 3n$

$|Conf| = 2^{n^2}$ $T_{brute}(n)$ $2^{n^2} C(n) \neq O(2^n)$

$(n \times n)^n$ $(n \times n)^1$ $(n \times n)^3$

Heuristics:

- early stopping
- choose to focus on likely good choices next.
- memoizing (avoid repeating computations)
- symmetry



Subset Sum

ex arr = [1, 5, 5, 10, 15, 20]

target = 25

?

While below look for next.

```
ssum(arr, tgt):          // decision.
    if |arr| = 0:
        return True if tgt = 0
               False otherwise.
    let a, rest = arr
    // a ∈ solution
```

```
if ssum(rest, tgt - a):
// a/k/a solution   return True
elif ssum(rest, tgt)
    return True
return False
```

$T_{ssum}(n) = O(2^n)$ Same proof as Hanoi

Memoize:

$$fib(n) = \begin{cases} 1 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ f(n-1) + f(n-2) & \text{otherwise} \end{cases}$$