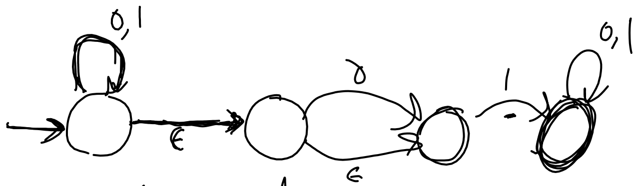
NFAs

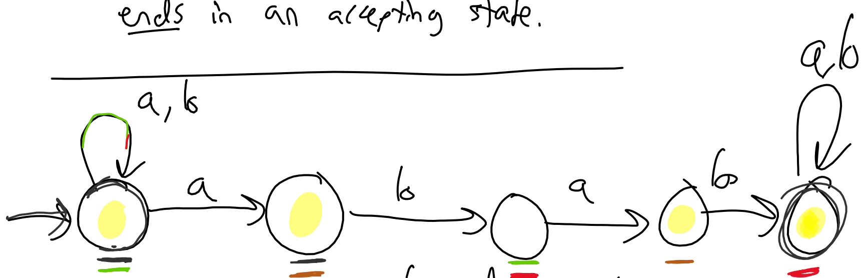
Thursday, January 30, 2020 3:32 PM



✓ $(0+1)^* 1 (0+1)^*$?

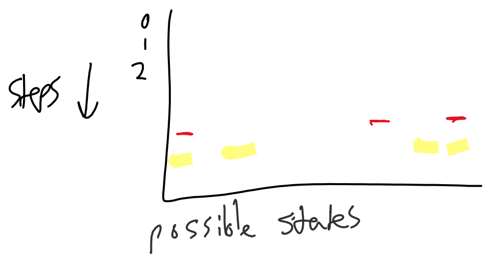
— "take any possible transition for the next symbol. And ε is a free move"

— "An NFA "accepts" a string iff ~~all?~~ any? Some path following above rule ends in an accepting state.

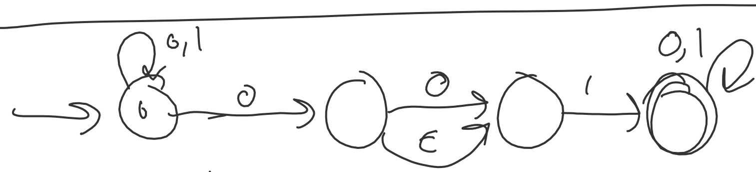


ababa ✓

$(a+b)^* abab (a+b)^*$



$2^5 = 32$ possible ways to fill each row



$(0+1)^* 0 (0+\epsilon) 1 (0+1)^*$

Interpretations of Nondeterminism

many threads/cores interpretation

how many? there could be a lot

- (at least for finite automata)....

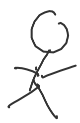
How many states could he be in?

$$\left|\mathbb{P}(Q)\right| = 2^{|Q|}$$

$\mathbb{P}(Q)$ also written as 2^Q

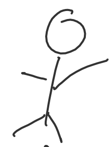
$2^{|Q|}$ may be very large compared to $|Q|$ but it's still finite

- prove and verify



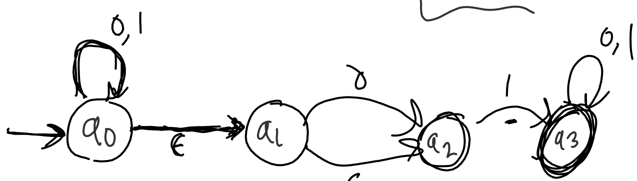
"N accepts "abababab..." "

"I'll show you...



prover verifier

$$\rightarrow q_0 \xrightarrow{0} q_0 \xrightarrow{\epsilon} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_3 \xrightarrow{0} q_3 \checkmark$$



✓ $(0+1)^* 1 (0+1)^*$?

$\delta(q_1, 1) \rightarrow \emptyset$

0 0 1 0

- ... many more...

Are all DFAs also NFAs?

Yes ✓

Are ϵ-transitions necessary?

No,

Formal notation

DFAs = $(Q, \Sigma, \delta, s, A)$

NFAs = $(Q, \Sigma, \delta, s, A)$

for dfa, $\delta : Q \times \Sigma \to Q$

for nfa, $\delta : Q \times (\Sigma \cup \{\epsilon\}) \to \mathcal{P}(Q)$

a subset of states

for dfa: $\delta^* : Q \times \Sigma^* \to Q$

$\delta^* : Q \times \Sigma^* \to \mathcal{P}(Q)$

$\underline{\delta}^* : \mathcal{P}(Q) \times \Sigma^* \to \mathcal{P}(Q)$

$\epsilon\text{-reach} : Q \to \mathcal{P}(Q)$ "all states reachable by ϵ-transitions"

$\epsilon\text{-reach}(q) = \delta(q, \epsilon)$

$\underline{\epsilon\text{-reach}} : \mathcal{P}(Q) \to \mathcal{P}(Q)$ "all states reachable from any of the states in S taking ϵ transitions"

$$\underline{\epsilon\text{-reach}}(S) = \bigcup_{q \in S} \left(\epsilon\text{-reach}\ q \cup \{q\} \cup \bigcup_{p \in \epsilon\text{-reach}(q)} \underline{\epsilon\text{-reach}}(p) \right)$$

equiv $\bigcup S$

a set of states

$\underline{\delta}^* = \ldots$ to fill in see course notes

Thompson's Alg.

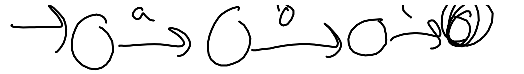
Regular language to an NFA.

Regexp:

$\emptyset$

ϵ

$\{abc\}$



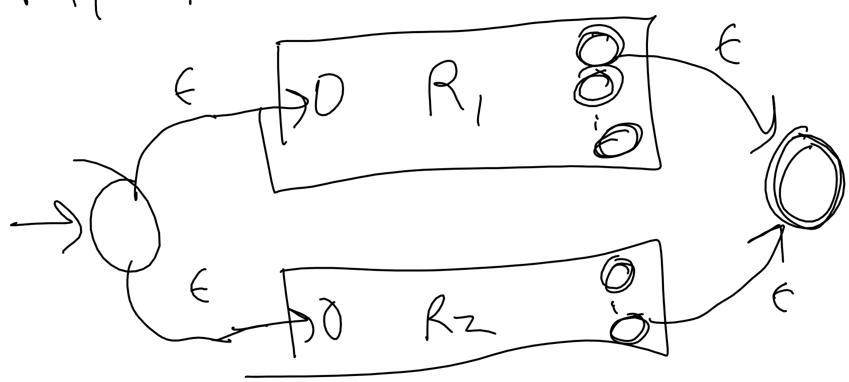
$R_1 \cup R_2$

R_1^*

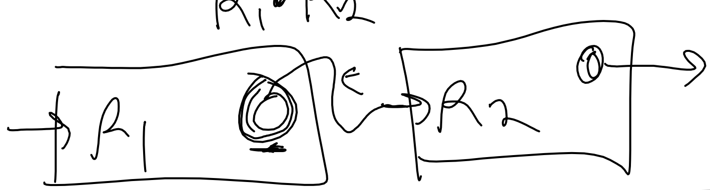
$R_1 \circ R_2$

$R_1 \cap R_2$ by prod. construction.

$R_1 \cup R_2$



$R_1 \circ R_2$



R^*