Regular Languages and Expressions

Lecture 2

Friday, January 24, 2020

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Part I

Regular Languages

Regular Languages

A class of simple but useful languages.
The set of regular languages over some alphabet Σ is defined inductively as:

1. $\emptyset$ is a regular language.
2. $\{\epsilon\}$ is a regular language.
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5. If L_1, L_2 are regular then $L_1 L_2$ is regular.
6. If L is regular, then $L^* = \cup_{n \geq 0} L^n$ is regular.
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Regular languages are closed under the operations of union, concatenation and Kleene star.

Some simple regular languages

Lemma

If w is a string then $L = \{w\}$ is regular.

Example: $\{aba\}$ or $\{abbabbab\}$. Why?

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Lemma

Every finite language L is regular.

Examples: $L = \{a, abaab, aba\}$. $L = \{w \mid |w| \leq 100\}$. Why?

More Examples

- $\{w \mid w$ is a keyword in Python program$\}$
- $\{w \mid w$ is a valid date of the form mm/dd/yy$\}$
- $\{w \mid w$ describes a valid Roman numeral$\}$
  $\{I, II, III, IV, V, VI, VII, VIII, IX, X, XI, \ldots\}$.
- $\{w \mid w$ contains "CS374" as a substring$\}$.

Part II

Regular Expressions

Regular Expressions

A way to denote regular languages

- simple patterns to describe related strings
- useful in
  - text search (editors, Unix/grep, emacs)
  - compilers: lexical analysis
  - compact way to represent interesting/useful languages
  - dates back to 50's: Stephen Kleene
    who has a star names after him.

Inductive Definition

A regular expression $\mathbf{r}$ over an alphabet $\mathbf{\Sigma}$ is one of the following:
Base cases:

- $\emptyset$ denotes the language $\emptyset$
- ϵ denotes the language $\{\epsilon\}$.
- $\mathbf{a}$ denote the language $\{a\}$.

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Inductive cases: If $\mathbf{r_1}$ and $\mathbf{r_2}$ are regular expressions denoting languages R_1 and R_2 respectively then,

- $(\mathbf{r_1} + \mathbf{r_2})$ denotes the language $R_1 \cup R_2$
- $(\mathbf{r_1 r_2})$ denotes the language $R_1 R_2$
- $(\mathbf{r_1})^*$ denotes the language R_1^*

Regular Languages vs Regular Expressions

Regular Languages

$\emptyset$ regular
$\{\epsilon\}$ regular
$\{a\}$ regular for $a \in \Sigma$
$R_1 \cup R_2$ regular if both are
$R_1 R_2$ regular if both are
R^* is regular if R is

Regular Expressions

$\emptyset$ denotes $\emptyset$
ϵ denotes $\{\epsilon\}$
$\mathbf{a}$ denote $\{a\}$
$\mathbf{r_1} + \mathbf{r_2}$ denotes $R_1 \cup R_2$
$\mathbf{r_1 r_2}$ denotes $R_1 R_2$
$\mathbf{r}^*$ denote R^*

Regular expressions denote regular languages — they explicitly show the operations that were used to form the language

Notation and Parenthesis

- For a regular expression $\mathbf{r}$, $L(\mathbf{r})$ is the language denoted by $\mathbf{r}$. Multiple regular expressions can denote the same language!
  Example: $(0 + 1)$ and $(1 + 0)$ denote same language $\{0, 1\}$

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  Example: $rst = (rs)t = r(st)$,
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  Example: $rst = (rs)t = r(st)$,
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- Superscript $+$. For convenience, define $\mathbf{r}^+ = \mathbf{r}\mathbf{r}^*$. Hence if $L(\mathbf{r}) = R$ then $L(\mathbf{r}^+) = R^+$.
- Other notation: $r + s$, $r \cup s$, $r|s$ all denote union. rs is sometimes written as $r \bullet s$.

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- Given a language L "in mind" (say an English description) we would like to write a regular expression for L (if possible)
- Given a regular expression $\mathbf{r}$ we would like to "understand" $L(\mathbf{r})$ (say by giving an English description)

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- $(\epsilon + 1)(01)^*(\epsilon + 0)$: alternating 0s and 1s. Alternatively, no two consecutive 0s and no two consecutive 1s

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- $\emptyset\mathbf{0}$: $\{\}$
- $(\epsilon+\mathbf{1})(\mathbf{01})^*(\epsilon+\mathbf{0})$: alternating $\mathbf{0}$s and $\mathbf{1}$s. Alternatively, no two consecutive 0s and no two consecutive 1s
- $(\epsilon+\mathbf{0})(\mathbf{1}+\mathbf{10})^*$: strings without two consecutive 0s.

Creating regular expressions

- bitstrings with the pattern **001** or the pattern **100** occurring as a substring

Creating regular expressions

- bitstrings with the pattern $\mathbf{001}$ or the pattern $\mathbf{100}$ occurring as a substring
  one answer: $\mathbf{(0+1)^*001(0+1)^* + (0+1)^*100(0+1)^*}$

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- bitstrings with an even number of $\mathbf{1}$'s
  one answer: $\mathbf{0^* + (0^*10^*10^*)^*}$
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  one answer: $\boldsymbol{r}\mathbf{1}\boldsymbol{r}$ where $\boldsymbol{r}$ is solution to previous part

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- bitstrings with an odd number of $\mathbf{1}$'s
  one answer: $r\mathbf{1}r$ where r is solution to previous part
- bitstrings that do *not* contain $\mathbf{01}$ as a substring

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- bitstrings with the pattern $\mathbf{001}$ or the pattern $\mathbf{100}$ occurring as a substring
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- bitstrings with an even number of $\mathbf{1}$'s
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- bitstrings with an odd number of $\mathbf{1}$'s
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  one answer:$\mathbf{1^*0^*}$

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- bitstrings with an odd number of $\mathbf{1}$'s
  one answer: $r\mathbf{1}r$ where r is solution to previous part
- bitstrings that do *not* contain $\mathbf{01}$ as a substring
  one answer:$\mathbf{1}^*\mathbf{0}^*$
- bitstrings that do *not* contain $\mathbf{011}$ as a substring

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- bitstrings with an even number of $\mathbf{1}$'s
  one answer: $\mathbf{0^* + (0^*10^*10^*)^*}$
- bitstrings with an odd number of $\mathbf{1}$'s
  one answer: $\boldsymbol{r1r}$ where $\boldsymbol{r}$ is solution to previous part
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  one answer:$\mathbf{1^*0^*}$
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  one answer:$\mathbf{1^*0^*(100^*)^*(1+\epsilon)}$

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  one answer: 1^*0^*
- bitstrings that do *not* contain $\mathbf{011}$ as a substring
  one answer: $1^*0^*(100^*)^*(1+\epsilon)$
- Hard: bitstrings with an odd number of 1s *and* an odd number of 0s.

Bit strings with odd number of **0**s and **1**s

The regular expression is

$$(\mathbf{00} + \mathbf{11})^* (\mathbf{01} + \mathbf{10}) \Big(\mathbf{00} + \mathbf{11} + (\mathbf{01} + \mathbf{10})(\mathbf{00} + \mathbf{11})^* (\mathbf{01} + \mathbf{10})\Big)^*$$

(Solved using techniques to be presented in the following lectures...)

Regular expression identities

- $r^* r^* = r^*$ meaning for any regular expression r, $L(r^* r^*) = L(r^*)$
- $(r^*)^* = r^*$
- $rr^* = r^* r$
- $(rs)^* r = r(sr)^*$
- $(r + s)^* = (r^* s^*)^* = (r^* + s^*)^* = (r + s^*)^* = \ldots$

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Question: How does on prove an identity?

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Question: How does on prove an identity?
By induction. On what? Length of r since r is a string obtained from specific inductive rules.

A non-regular language and other closure properties

Consider $L = \{0^n 1^n \mid n \geq 0\} = \{\epsilon, 01, 0011, 000111, \ldots\}$.

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Theorem

L *is* **not** *a regular language.*

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How do we prove it?

Other questions:

- Suppose R_1 is regular and R_2 is regular. Is $R_1 \cap R_2$ regular?
- Suppose R_1 is regular is $\bar{R}_1$ (complement of R_1) regular?