Describe deterministic finite-state automata that accept each of the following languages over the alphabet $\Sigma = \{\texttt{0}, \texttt{1}\}$. You may find it easier to describe these DFAs formally than to draw pictures.

Either drawings or formal descriptions are acceptable, as long as the states Q, the start state s, the accept states A, and the transition function δ are all clear. Try to keep the number of states small.

1. All strings in which the number of `0`s is even **and** the number of `1`s is *not* divisible by 3.

2. All strings in which the number of `0`s is even **or** the number of `1`s is *not* divisible by 3.

3. Given DFAs M_1 and M_2, all strings in $\overline{L(M_1)} \oplus L(M_2)$.
   Recall that for two sets A and B, their symmetric distance $A \oplus B$ is the set of elements in either A or B, but not both.

Work on these later:

4. All strings that are **both** the binary representation of an integer divisible by 3 **and** the ternary (base-3) representation of an integer divisible by 4.

   For example, the string `1100` is an element of this language, because it represents $2^3 + 2^2 = 12$ in binary and $3^3 + 3^2 = 36$ in ternary.

5. All strings in which the subsequence `0101` appears an even number of times.

6. All strings w such that $\binom{|w|}{2} \bmod 6 = 4$. *[Hint: Maintain both* $\binom{|w|}{2} \bmod 6$ *and* $|w| \bmod 6$.*]*

*7. All strings w such that $F_{\#(\texttt{10}, w)} \bmod 10 = 4$, where $\#(\texttt{10}, w)$ denotes the number of times `10` appears as a substring of w, and F_n is the nth Fibonacci number:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$